



$$E = MD^2$$

Excellence in Maths Education through (E-)Debate and Diversity

Ebook





Contents

Introduction	3
Online Survey: Conclusions.....	6
Teachers.....	6
Pupils.....	7
Parents.....	11
Online Debates for the Exchange of Experiences	12
Annex 1 – Good Practices.....	14
References	62



Introduction

Scientists have long used maths to describe the physical properties of the universe, some of whom have even claimed that the universe itself is math, including human beings. Most people are aware of maths' ubiquity in our everyday lives, yet we very often fail to make proper use of it. This is something that happens every day in the classroom.

The most important educational period of our lives is between the ages of 4 and 15, during which time, pupils create learning mechanisms, build knowledge and develop basic skills. However, the results of international knowledge and skills tests do not necessarily show that schools are helping pupils acquire and develop these basic skills. According to PISA results from 2018, pupils from Spain, Cyprus, Romania, and Croatia achieved a minimum of Level 2 in maths, while pupils from North Macedonia did not reach this basic level. Moreover, tests carried out in different European countries are beginning to show the devastating impact of the COVID-19 pandemic on schoolchildren's performance, specifically on the most vulnerable.

The biggest cause of concern for disengaged or lower-performing pupils who, according to Schleicher (2019), will fade away to the margins of



society without the right education. Several academic papers and global reports, such as the Eurydice Network's report: "Mathematics Education in Europe: Common Challenges and National Policies", point to the importance of learner motivation and engagement. In light of this, the project "E=MD2: Excellence in Maths Education in inclusive classroom through e-Debate and Diversity" was designed with the aim of improving the mathematical skills of pupils across Europe, especially those with specific needs that affect their performance in maths, such as dyscalculia, dysgraphia and mathematics anxiety, by increasing their motivation to learn and making their experience less passive.

That being said, the E=MD2 project working team believes that pupils should be asked which aspects of accessing or understanding mathematical content they find challenging. Mathematical debates are a popular method that allow pupils to exchange ideas and viewpoints about learning maths, resulting in a better understanding of each other's experiences. With this in mind, the E=MD2 working team set out to ask pupils, teachers and parents their opinions about teaching and learning mathematics.

Between September and December 2022, the E=MD2 team conducted a series of surveys and online debates in several European countries. On the one hand, 177 teachers, 274 pupils and around 100 parents from Spain, Cyprus, Romania, Croatia, and North Macedonia participated in an online



survey where they described both the realities and challenges of teaching and studying maths. On the other hand, 20 parents and 20 teachers from two schools in Romania and North Macedonia participated in an online debate to exchange experiences, developing a better understanding of the challenges that pupils face at school and at home when engaging in learning material. This was done in the hope of better supporting and motivating pupils to improve their performance. **This document is a summary of the findings of both the surveys and the debates.**

Finally, as part of the mission of E=MD2 to provide inclusive teaching of the highest standard, a fundamental part of this process is supporting maths teachers, the E=MD2 team compiled a collection of maths activities to be implemented in class as part of E=MD2's new teaching method, as well as creating the interactive [e- MATH DEBATE platform](#) for teachers, pupils and parents.





Online Survey: Conclusions

Teachers

The aim of the E=MD2 project is to find out where the problems lie in relation to the teaching and understanding of the maths curriculum. The first method used to obtain data was an online survey in which more than 500 people, including teachers, participated. The questions were designed to learn more about their experiences in the classroom and their opinions on the curriculum, their attitudes towards pupils and the pupils' attitudes towards mathematics.

Below are the most relevant findings:

Most teachers consider the topics of the maths curriculum that they teach to be appropriate (64.73%). However, the vast majority of them claim that the curriculum contains too much content (76.27%), which prevents them from attending to the needs of pupils with specific needs (68.93%) and/or devoting time to gifted pupils (69.49%). More importantly, most teachers in the sample (79.66%) indicated that more maths lessons are required each week. This data highlights that teacher's work under great time pressure with an extremely compressed curriculum, making it difficult for them to deliver lessons in such a way that all pupils understand and can apply the knowledge.



In terms of teachers' perception and that of their pupils, almost all of the teachers interviewed agreed that mathematics is a difficult subject (76.84%) and that pupils struggle to see the point of it (71.19%). Another surprising finding was that even though the participants had 15 or more years of teaching experience, only a small proportion of them reported feeling competent enough to teach maths to pupils with specific needs (12.43% feel competent and 40.68% agree to feeling partially competent).

Despite this, it is clear that teachers do the best they can with what they have available. Almost all (98.03%) considered peer-to-peer learning to be a useful strategy and acknowledged that teaching is a great way to learn (96.61%), making this an opportunity for both proficient pupils and those with specific needs to interact and help each other.

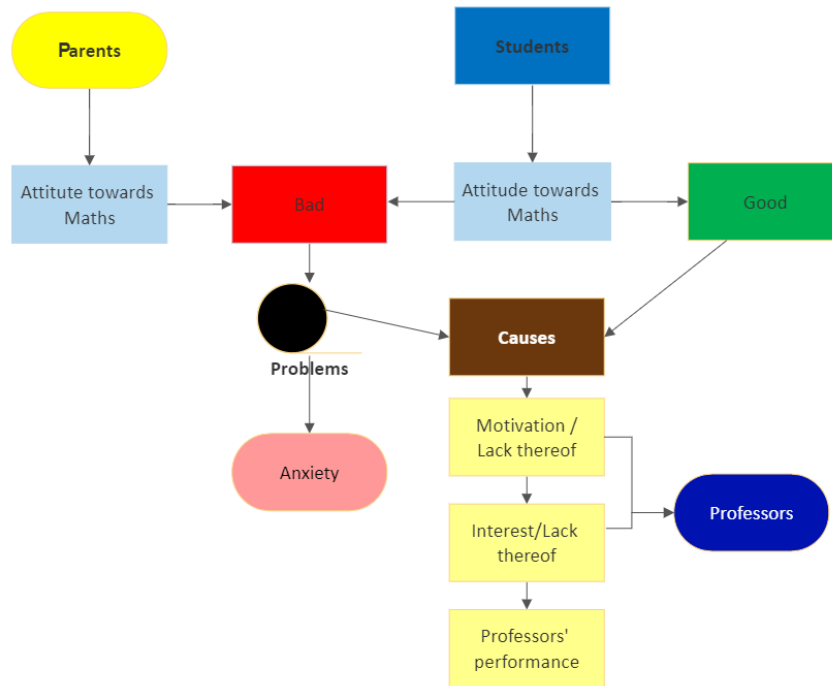
From this survey, we can see that teachers are aware of the problems in teaching maths but have no time to help their pupils and lack opportunities to improve their skills, which only perpetuates these issues.

Pupils

Before the survey, the working team of the E=MD2 project had the following hypothesis: pupils' negative attitudes towards maths is strongly influenced by both poor teaching and by their parents. When both attitudes and influences are negative, this leads to "maths anxiety". To the



surprise of the working team, the results of the survey are notably more nuanced than first anticipated.



Unexpectedly, of the 274 respondents, most pupils stated that they love or like maths (23.72% love and 48.9% like maths), and only 10.58% of pupils said that they disliked maths altogether. This rather positive tendency is also shown when pupils were asked if it would be "nice if there were no maths lessons", to which 58.42% disagreed. However, the survey also shows a significant lack of interest in maths among pupils, who, for example, find it neither interesting (50%) nor useful whatsoever (27.01%), which shows a clear 50:50 split among pupils.

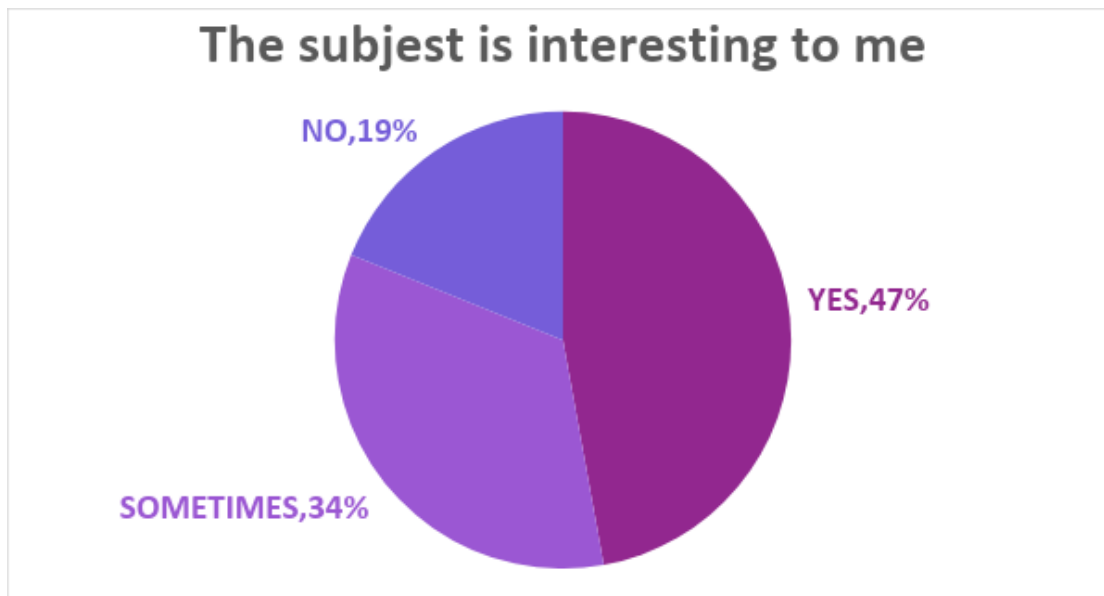


Image 5. Pupils' general opinion about maths

With regards to maths anxiety, 21.79% of the pupils agreed with the statement: "maths makes me nervous and confused", and the other 36.43% answered "sometimes". This negativity is also reflected in the fact that 77% of the pupils surveyed answered "yes" or "sometimes" to the following statement: "after the written exam, I feel that I made a lot of mistakes even in the tasks I knew".

Equally, it is important to acknowledge the role of teachers in providing a good working environment during maths lessons. Between 40% and 50% of all respondents feel that their classroom environment is relaxed, which is mainly due to the level discipline. Only 15% of pupils do not feel relaxed about their working environment and the level of discipline in class, the latter of which correlates with teachers' seniority.

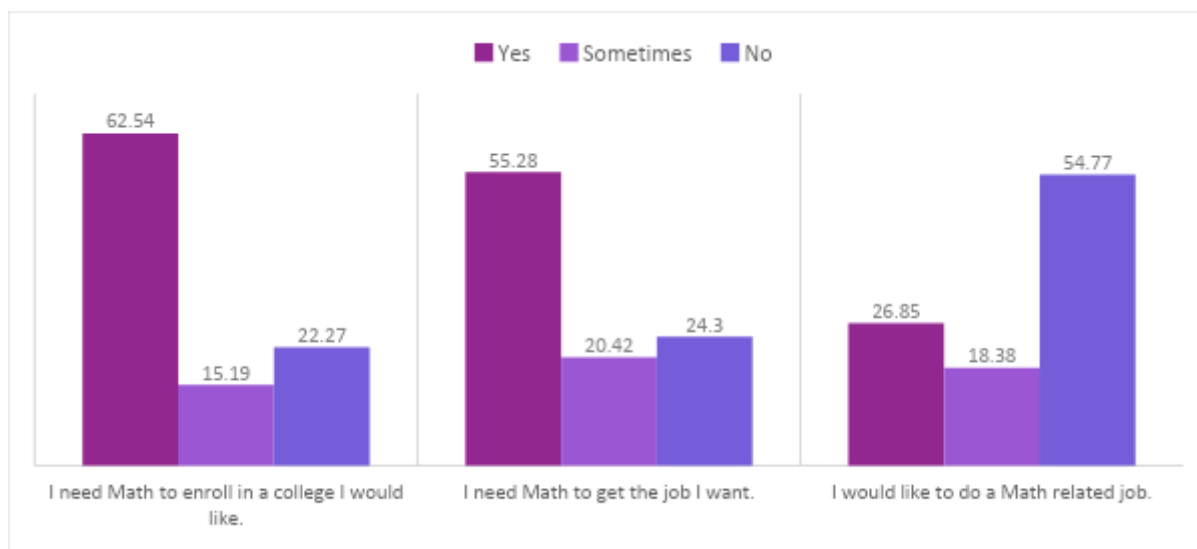


Image 9. General opinion of pupils on the need for maths in later life

Even though teachers perceive maths to be a difficult subject, they invest time in finding interesting teaching methods to familiarise pupils with the material, one of which is peer-to-peer work. In this regard, 37.46% of the pupils surveyed agreed with the statement: "I love when my peers explain tasks to me so that I learn better" and 45% of the pupils said that they enjoy explaining maths problems to their classmates, which they believe, helps them to learn better. As expected, pupils learn better in pairs, through discussions, and by participating in the teaching process.

As the survey shows, teachers see mathematics as a difficult subject, therefore they seek the help of pupils that have an interest in maths and can understand it easily. These pupils act as teachers and help other pupils to understand the material. However, there is still a lot of confusion and anxiety towards maths when pupils tackle problems by themselves, for example in an exam. This can result in a lack of motivation, which translates



to a general disinterest in mathematics among almost half of pupils across Europe. Furthermore, these results do not take into account the fact that an average of two or three pupils in each class have specific needs (Butterworth, B. & Kovas, Y., 2013). Due to an already packed curriculum and time constraints in the classroom, these pupils do not receive much needed personal attention from their teachers.

Parents

As for parents, the general opinion is that maths is a difficult subject and that pupil's lack the motivation to learn. In fact, from the 169 respondents, 61.57% admitted that their child struggles with maths. The reasons given by parents are lack of motivation and poor understanding of the curriculum. Specifically, 40% of pupils do not feel motivated and almost 31% do not understand the subject according to their parents.

When asked if their attitude towards mathematics could influence their child's attitude, opinions are divided almost 50:50 between those who believe that this may be the case and those who do not. The perception among Croatian parents is that their attitude has no influence on that of their children, while Romanian, Macedonian and Spanish parents share the opposite opinion.

When asked about the things that could help their child to feel more motivated when studying maths, the general response was that maths should be more interesting, that there should be more "gamification" in



math classes. Moreover, it was viewed as important for pupils to see a connection between daily life and the need for maths; the same opinion was also expressed by teachers in the online debates. The conclusions drawn from these are discussed in the following section of this document.

Online Debates for the Exchange of Experiences

As previously mentioned, the E= MD² work team set out to gain a better understanding of the challenges that pupils face at school and at home when studying maths. With this in mind, two online debates were organised for parents and teachers to share their experiences.

On 3 November 2022, a group of ten teachers and ten parents from North Macedonia met for an online debate. The debate was moderated by staff from the school OOU "ILINDEN" Kriva Palanka. On 12 November 2022, a similar debate took place at "Ion Agârbiceanu" secondary school in Alba Iulia, Romania, involving ten maths teachers and ten parents. In both debates, parents and school staff discussed the possible ways in which they could support and motivate pupils to improve their performance.

The conclusions drawn in North Macedonia and Romania are as follows: Although mathematics primarily teaches pupils critical thinking skills, gives them self-confidence and opens up numerous possibilities for the future, the extremely dense curriculum, (which in Romania is very theory-based),



prevents teachers from providing practical lessons. Furthermore, the number of maths lessons per week does not allow teachers to work with pupils who have higher abilities or those with specific educational needs.

According to parents and teachers, the lack of interest in maths becomes apparent, as pupils get older, as many of them initially enjoy the subject. Consequently, as lessons becomes more complex, pupils have less independence and feel less confident in their abilities. This becomes apparent when pupils need help to answer questions on their homework, which often leads to frustration when parents try to help but end up misguiding them.

Overall, parents asked teachers to make maths more tangible for their pupils as currently, many struggle to understand the usefulness of maths because it is so abstract. Therefore, they would like more practical exercises that reflect and can be useful in everyday life.



Annex 1 – Good Practices

Linear Programming for Gifted Pupils aged 12-14

These activities aim to help pupils understand some of the differences between linear algebra and linear programming, using graphs of first-degree functions and solving inequalities. They then practice what they have learnt by completing exercises, including ones that can be applied to daily life.

Topic

Algebra

Learning Outcomes

This learning plan provides activities that guide/enable/support pupils (chiefly those that are already proficient in maths) to:

- Identify the best connections between the first-degree function and the graphical representation of the function.

- Work out the solution to a minimum or maximum problem.
- Identify the graphical solution of a minimum or maximum problem.
- Calculate the intersections of the graph with the coordinate axes.



How does it work?

The learning plan suggests activities that encourage pupils to:

- Explore the lines of connection between a function's field of definition and its graphic representation.
- Look for the shortest connections.
- Specify expected conditions.
- Clarify whether the graph provides solutions for

determining the required minimum or maximum (optimised) quantities.

- Graphically represent the conditions in the hypothesis of the problem.
- Use their newly acquired knowledge to determine the maximum or minimum value of the function.
- Explore how to determine the requirements of the problem with the help of a graph.

Why is it a good way to practice?

It encourages critical thinking, creativity and communication, working with content that is connected to the real world.

Assessment

Pupils use the internet to find simple or more advanced real-world problems and attempt to solve them using linear programming.

The emphasis on real-world problems makes the INNOMATH project especially relevant as part of the Mathematics Meets Industry Day. This is



reflected by the educational potential of this activity in particular, as well as that of the other activities included in the project.

Inclusion

The approach caters to the needs of more gifted pupils.

Resources

Maths textbook (Year 8 or equivalent), problem collections:

<https://www.khanacademy.org/math>

Language

English



Services Assessment in an Organisation

The learning plan provides activities for pupils that involve them in a hands-on research process that can be applied to service evaluation. Pupils are taught basic principles and stages of research processes, from expressing the problem and deciding on the goal of their research, to the final presentation of the results and conclusions.

Topic

Statistics (Research Methodology)

Learning Outcomes

This learning plan offers activities that guide/enable/support a pupil to acquire the following skills:

- Methods of Data Collection and Sampling Methods and Techniques.
- Design and use of appropriate questionnaires (printed or online).
- Considering questionnaire validity and reliability control methods (use of appropriate software).
- Using methods of statistical analysis and presentation of results (use of appropriate software).
- Presentation of results - writing a detailed research report.



How does it work?

Teachers and pupils participate in a real-life process of evaluating a community service provider that the pupils are interested in. Pupils review or propose ways in which the services provided could be improved for the benefit of the company and for the public who use them, with the relevance of the pupils' conclusions in this activity being a source of motivation. On the other hand, the skills acquired through this research project are very important in the modern job market. These skills are acquired through making contact with the service provider's representatives to carry out consultations, but also through teamwork in preparing questionnaires (printed and online), data collection and input, data analysis, presentation of results and drawing conclusions. Support and evaluation from teachers should be available throughout the process, with feedback given to the pupils at each stage.

Why is it a good practice?

It encourages critical thinking, creativity and communication, and the content links back to the real world. Moreover, it provides opportunities for debate and discussion and the use of digital means.

Assessment

Feedback and evaluation are continuous, from the preparation of the relevant questionnaires to carrying out research and preparing and analysing results.



Inclusion

The entire approach provides material that is appropriate for all pupils, meaning that it can be used in a mixed ability class.

Resources

The theoretical framework will be taught in class. However, pupils will process the questionnaires (printed or online), enter data, analyse statistics and then prepare the presentation of their results using ICT facilities (with the support of teachers).

Resources, Tools, Material, Attachments, Equipment

Important Factors for Evaluating Services:

<https://www.fotoinc.com/news-updates/3-ways-to-evaluate-your-services>

<https://www.qualtrics.com/blog/how-to-measure-service-quality/>

Data Collection Methods and Sampling Methods and Techniques:

<https://www.slideshare.net/swatiluthra5/sampling-ppt>

<https://www.slideshare.net/7mukut/sampling-techniques-49115431>

<https://www.slideshare.net/Indraneeltu/icfai-ib>

Creation and use of appropriate questionnaires (Printed or Online):

<https://www.slideshare.net/deepthisreenivas1/questionnaire-design-in-research>

<https://mopinion.com/top-21-best-online-survey-software-and-questionnaire-tools-an-overview/>

Questionnaire Validity and Reliability Control Methods (Use of Appropriate Software):



<https://www.youtube.com/watch?v=Odlhhtg-3LE>

<https://www.youtube.com/watch?v=pVfByfoQ1IU>

Methods of Statistical Analysis and Presentation of Results (Use of appropriate Software):

Presentation of results: writing a detailed research report

Language

English



Deriving Formulae and Determining the Area of 2D Shapes Drawn on a Grid of Dots

The learning plan provides for activities for pupils that are involved in a real research process with application in services evaluation. Basic issues and stages of the research process are taught, from the formulation of the problem and the goal to the final presentation of the results and conclusions.

Algebra

Learning Outcomes

By the end of the lesson, the pupil will be able to:

- Express a variable through other variables in a given equation.
- Perform simple formulae.
- Determine the area of 2D shapes drawn on a grid of dots.
- Use formulae from maths and other subjects.
- Present concise, substantiated arguments to explain solutions or generalisations using symbols, diagrams, or graphs.
- Develop a sense of cooperation and teamwork with classmates.



How does it work?

40-minute lesson plan:

Activity 1: Class discussion about prior knowledge on calculating the area of regular shapes.

A shape consisting of a rectangle and two semicircles is shown to the class and pupils are asked how to find the calculate its area using formulae.

Pupils explain that the semicircles should be merged to form a circle and that the total area of the shape is the sum of the areas of the rectangle and the circle, with the formula to calculate this being:

$$P = w^2 \cdot \pi / 4 + x \cdot w$$

Activity 2: Pupils work in pairs to look for a connection between the area of shapes drawn on a dotted grid and the coordinates of their vertices, for example:

Express the formula for Area A of the shape drawn on the dotted grid (Pick's theorem).

The teacher asks the following questions:

How would you write down your findings?

Do you notice any patterns?



Can you find a general rule?

The area of the shape is one less than the sum of its interior vertices and half of its exterior vertices. The formula is as follows:

$$A = I + p / 2 - 1 \text{ (known as Pick's theorem).}$$

In pairs, pupils draw shapes on a dotted grid and then count the shapes' interior and exterior vertices, and then calculate their areas using Pick's theorem.

Pupils' self-assess and discuss their answers in pairs.

Pupils use the *GeoGebra* app in pairs to draw shapes and add the coordinates of the interior and exterior vertices, calculating the area with Pick's theorem and checking their answers in the Area Calculation Menu.

Activity 3: In the schoolyard, pupils create a shape using cones and some string. They then add up the coordinates of the interior and exterior vertices and calculate the shape's area using Pick's theorem.

Why is it a good practice?

The above activities initiate:

- Creativity
- Communication
- Critical thinking skills
- Cooperation

Assessment



These skills equip pupils to learn independently. The first activity allows teachers to assess pupils' prior knowledge of the topic which helps them to plan future activities. The activity that incorporates Pick's theorem helps pupils to connect what they are learning with real-life situations, develop critical thinking and communication skills, as well as promoting creativity and cooperation among peers. The use of digital tools to assess pupils' progress provides quick feedback for both pupils and teachers.

Inclusion

The teacher can add more kinaesthetic tasks, such as 2D shapes made from coloured paper that pupils move around a geometric board.

Resources

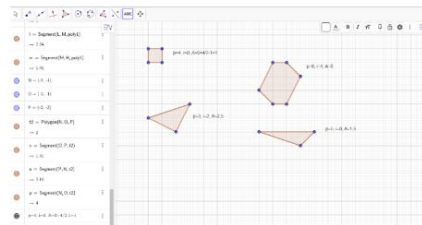
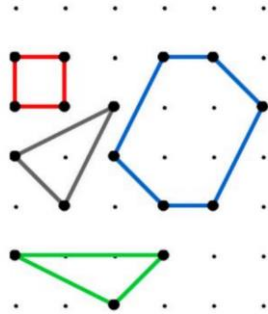
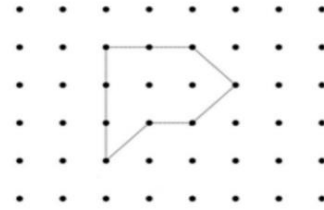
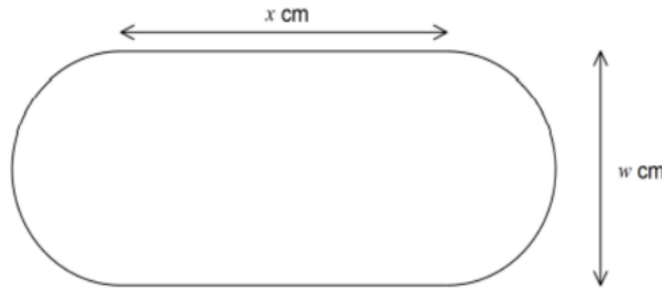
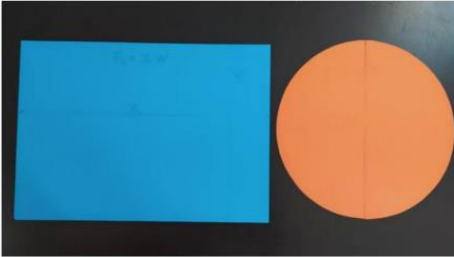
<https://nrich.maths.org/content/id/8506/1>

<https://nrich.maths.org/1867>

<https://www.geogebra.org>

Language

English





Percentages

Pupils will learn how to calculate a percentage of a given whole number and solve problems relating to increasing and decreasing percentages. The lesson aims to build on the topic of "Percentages" that is taught in the Autumn Term. It lasts for three lessons, or a duration of 120 minutes.

- Lesson 1 (40 minutes): Assessment of prior knowledge.
- Lesson 2 (40 minutes): Working with percentage increases and decreases.
- Lesson 3 (40 minutes): Reflection.

Topic

Percentages

Learning Outcomes

By the end of the lesson, pupils will be able to:

- Use simple fractions and percentages to compare quantities (visualisation).
- Create and solve problems involving percentage increases and decreases.
- Research and use various strategies in solving problems within texts.
- Develop a sense of cooperation and teamwork with classmates.



How does it work?

Lesson 1: Diagnostics for Calculating the Percentage of a Number

Fold the Shape by X Percent (5 minutes)

In this short activity, pupils are divided into four groups and each group receives a different worksheet. Pupils fold shapes according to the instructions on their worksheets, aiming to visually compare fractions and percentages of the shape.

Percentage Cards (35 minutes)

This activity allows pupils to practice calculating a percentage of a given whole number. Each pupil receives a card that includes one question and the solution to a task from another card. When a pupil reads the problem from their card, the others try to work out the answer. The pupil who has the solution to that problem then reads out the question on their card. The game continues until all of the tasks from the cards have been read out.

Lesson 2: Expanding the Term Percentage with Percentage Increases and Decreases

Stationary Shop (40 minutes)

Pupils are divided into four groups and the teacher sets up two imaginary stationary shops; two groups will be buyers and two groups will be sellers.



Pupils that are selling lay out classroom supplies (textbooks, notebooks, pencils, rulers...) for the other pupils to buy. The sellers then determine the price of those products and whether their price will increase or decrease by a certain percentage. Pupils write down the price of the item and the percentage increase, decrease and place them next to each product. Pupils who are buying choose one product at a time and need to calculate how much it will cost after the increase/decrease. The groups then swap roles.

Why is it a good practice?

The above activities initiate the development of critical thinking and communication skills, teamwork and creativity. These skills equip pupils to learn independently.

Assessment

The first activity of Lesson 1 allows teachers to assess pupils' prior knowledge of the topic, which helps them to plan future activities. The Stationary Shop activity in Lesson 2 allows pupils to connect what they are learning with real-life situations, develop critical thinking and communication skills, as well as promoting creativity and cooperation among peers.

The use of digital tools to assess pupils' progress provides quick feedback for both pupils and teachers.



Lesson 3: Evaluation

Game: Self-Evaluation using *Genially* (15 minutes)

Description: In pairs, pupils use their tablets or smartphones to access *Genially* to solve a quiz on percentage increases and decreases.

The quiz is designed so that pupils can immediately see where they went wrong and have a short discussion with their partner about their mistakes.

Quiz using *H5P* (20 minutes)

Pupils solve a quiz on *H5P*'s learning platform where pupils can view their points total. The quiz can be done online via the following link, in which case pupils will be able to send their answers to the teacher:
<https://ucimatematika.weebly.com/good-practice-percent.html>

Feedback (5 minutes)

A brief discussion about the quiz on *H5P* and things that the pupils feel that they did well.

- Formative Evaluation by monitoring pupil activity throughout all three hours and the self-evaluation with a quiz on *Genially*.
- Summative Evaluation is calculated through points scored on the *H5P* quiz.



Inclusion

Activity 1: Shapes drawn with an overlap of 50%, 25%, 20% or 10%, assessed by the teacher.

Activity 2: Lego cubes that are worth either 5% or 10% used to assemble 3D shapes.

Resources

Shapes for percentage of numbers & Percentage cards:

<https://ucimatematika.weebly.com/active-learning.html>

Percentage of numbers-Quiz in *Genially*:

<https://view.genial.ly/620226b64cdd0a00183fbe5d/game-percentage-of-number>

Language

English and Macedonian



50% of 16	35	20% of 85	8
25% of 160	17	1% of 100	40
1% of 200	1	25% of 184	2
5% of 140	46	5% of 100	7
20% of 55	5	20% of 115	11
10% of 120	23	20% of 0	12
50% of 96	0	25% of 148	48
5% of 200	37	10% of 160	10
25% of 60	16	15% of 20	15
5% of 180	3	25% of 140	9



Sale Discount 5% nonyet

Sale discount 20% nonyet

Sale discount 15% nonyet

Sale discount 25% nonyet

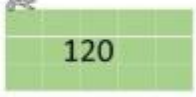
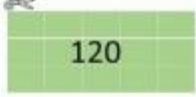
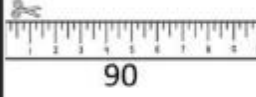
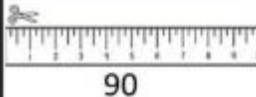




Sale discount 75% nonyet

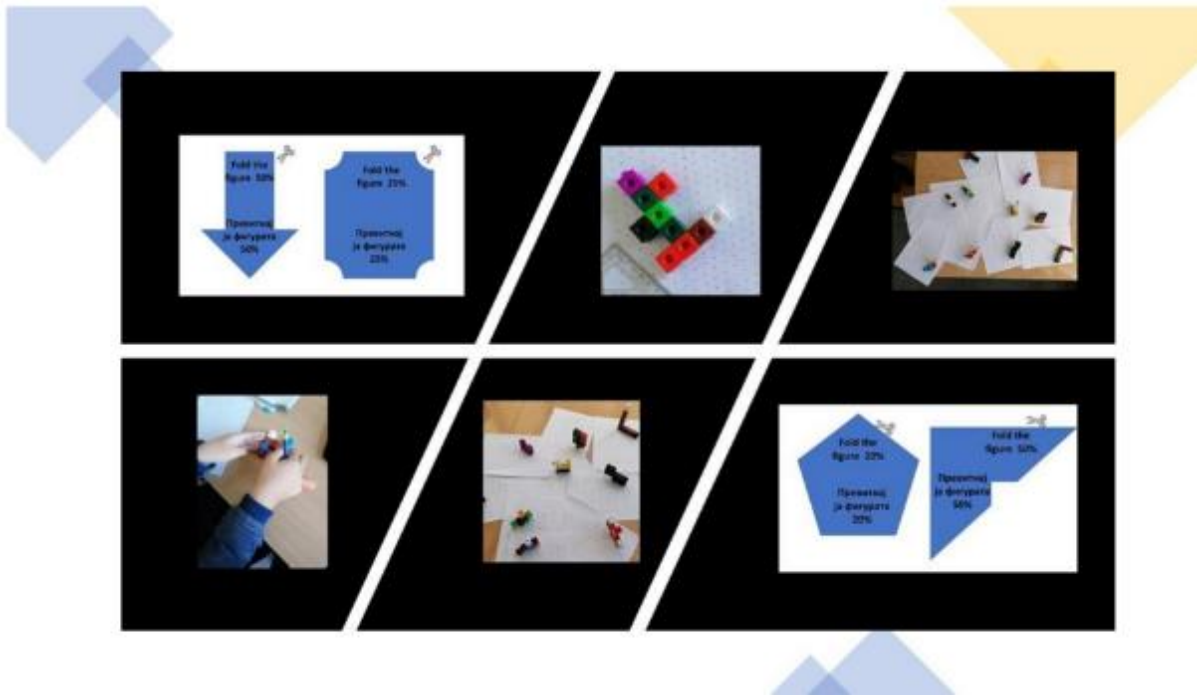
higher prices 22% nonyet

higher prices 44% nonyet

higher prices 25% nonyet



<p>Group 1</p>  <p>By folding show $\frac{1}{5}$</p> <p>If the figure is number 120, what is its value after folding?</p> <hr/>  <p>By folding show 20%</p> <p>If the figure is number 120, what is its value after folding?</p>	<p>Group 4</p>  <p>By folding show $\frac{1}{10}$</p> <p>If the figure is number 90, what is its value after folding?</p> <hr/>  <p>By folding show 10%</p> <p>If the figure is number 90, what is its value after folding?</p>
<p>Group 2</p>  <p>By folding show $\frac{1}{4}$</p> <p>If the figure is number 84, what is its value after folding?</p> <hr/>  <p>By folding show 25%</p> <p>If the figure is number 84, what is its value after folding?</p>	<p>Group 3</p>  <p>By folding show $\frac{1}{2}$</p> <p>If the figure is number 50, what is its value after folding?</p> <hr/>  <p>By folding show 50%</p> <p>If the figure is number 50, what is its value after folding?</p>





Learn Functions and discuss how Equations and Inequations can be solved using GeoGebra Representation

Solving inequations, especially non-linear ones, can be a long and difficult process. Using *GeoGebra*, pupils can represent non-linear functions and easily interpret the regions where the functions are positive or negative, helping them to solve the inequation.

Topic

Algebra and Functions

Learning Outcomes

The games in this section are designed to both support and improve pupils' digital, numeracy and linguistic skills.

How does it work?

Pupils start by drawing non-linear functions in *GeoGebra* and discussing how they change. The teacher then provides an activity in which the pupils have to represent parabolas and find a connection between the shape and:

- Whether the term of highest degree is positive or negative.
- Changing the independent term.

Teacher can help pupils by asking:

- Do the branches of the parabola go up or down? Is this connected to changes in the expression of the function?



- Do you notice any patterns?
- Can you find a general rule?

Pupils then solve in equations using the representation of the parabola and analysing where it is positive or negative.

Finally, they discuss their answers within their group and the rest of their peers.

Why is it a good practice?

The above activities initiate the development of:

- creativity
- communication
- Critical thinking skills
- Cooperation

Assessment

The activities develop critical thinking and communication skills, as well as promoting creativity and cooperation among peers. The use of online resources to assess pupils' progress provides quick feedback for both pupils and teachers.

Inclusion

The activities allow pupils with and without specific needs to mix and interact with each other.

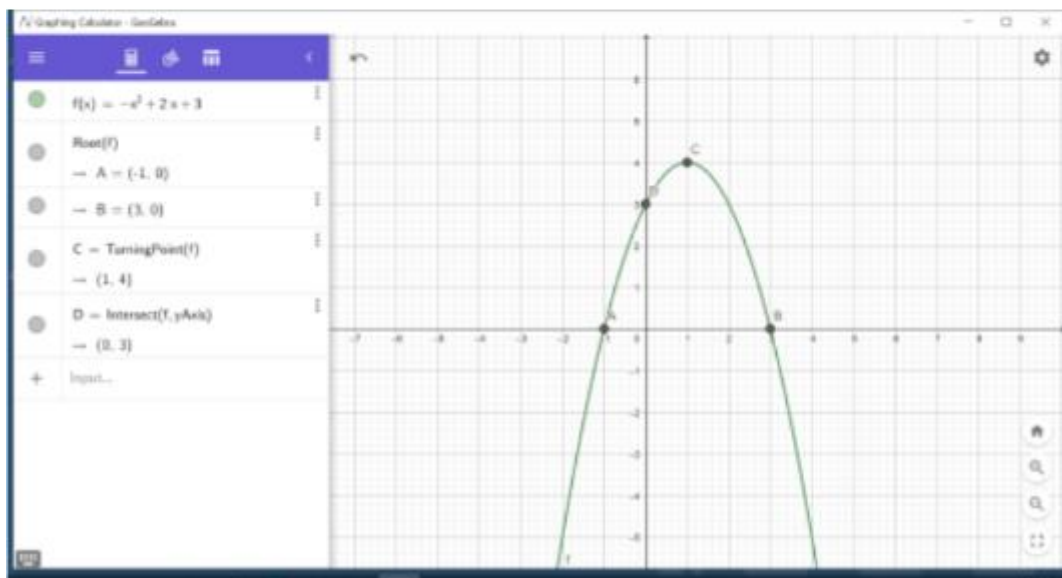
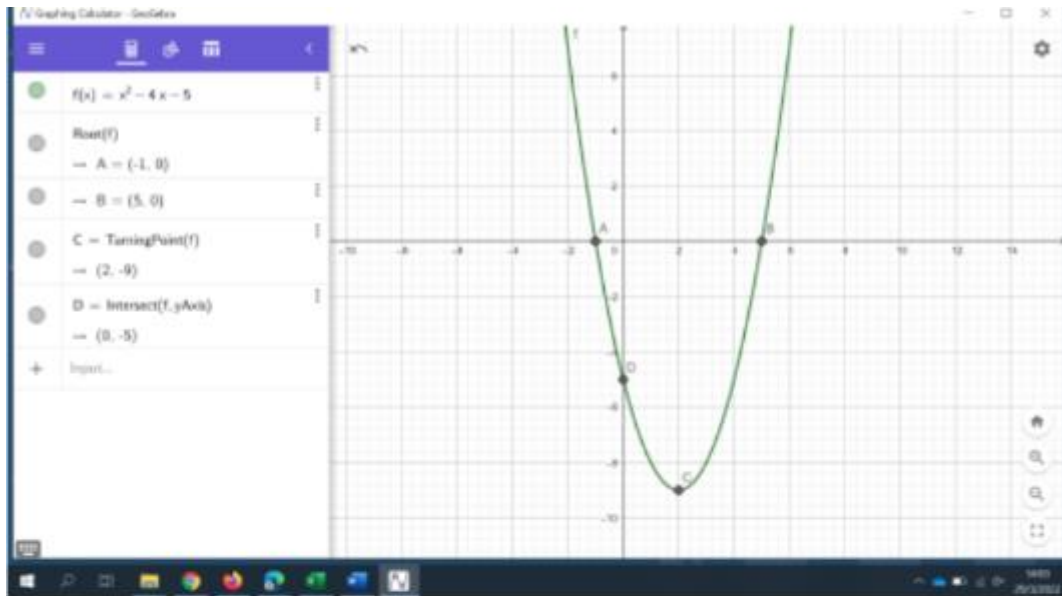
Resources

<https://www.geogebra.org/>

Language



English





Learn how to operate Integer Numbers (Addition, Subtraction, Multiplication and Division) with different variations and observing their different properties

Pupils sometimes find it challenging to calculate the different properties of operations that contain integers or minus numbers. Pupils can set the quiz to their required level of difficulty, self-correcting as they go and seeing where they need to improve.

Topic

Numbers

Learning Outcomes

The games included in this section are designed to support and improve digital, numeracy and linguistic skills.

How does it work?

Using *Google Classroom*, teachers provide their pupils with a link to different exercises for them to complete within a week. The exercises can be personalised and set at different levels, so that all pupils are catered for.

Pupils can work on the exercises at home and have as many attempts as they need to complete the work. The aim of this exercise is to highlight their mistakes so that they can improve their score each time that they repeat the exercise.



Pupils post a screenshot in *Google Classroom* before the deadline in order to be evaluated. In *Google Classroom*, teachers can see how many pupils completed the activities, as well as their results and their mistakes; they will then base the next quiz on the mistakes that were made most frequently.

Pupils have a week to complete each quiz and they should upload a screenshot of their results online. The evaluation is not quantitative (from 0 to 10), which pupils feel is a more visual and understandable way of working.

Why is it a good practice?

The activities initiate the development of:

- Critical thinking skills
- Creativity
- Communication

Assessment

The use of online resources to assess pupils' progress provides quick feedback for both pupils and teachers.

Inclusion

The exercises can be personalised and set at different levels of difficulty in order to cater to a class of mixed abilities.

Resources

<https://www.thatquiz.org/>



Language

English

t h a t q u i z Teachers: login or [create an account](#) or [\[search\]](#) or [\[learn more\]](#)

Login/Email Password

integers	fractions	concepts	geometry
Arithmetic	Identify	Time	Triangles
Inequality	Arithmetic	Money	Shapes
Averages	Inequality	Measure	Geometry
Exponents	Averages	Place value	Points
Factors	Simplify	Graphs	Angles
Algebra	Probability	Sets	Number line
Calculus			Trigonometry

vocabulary		geography	
English	Spanish	Americas	Europe
French	German	Africa	Asia

science

Cells	Anatomy	Elements	Conversion
-------	---------	----------	------------

English Español Deutsch Français Italiano Português Català
 Euskara Magyar Polski Slovenščina Türkçe Ελληνικά Українська
 中文(简体) עברית

Have a test code?



Visual Representations

An evidence-based strategy to help pupils learn abstract maths concepts and solve problems is to use visual representations. As opposed to a picture or a detailed illustration, a visual representation, often called a schematic representation or diagram, is an accurate description of the mathematical quantities and relationships of a given problem. It aims to show pupils' understanding of the problem, helping them to solve it correctly. For example, in the photo on the right, a pupil uses a visual representation, in this case a pie chart, to learn about equivalent fractions. Even though teachers implement this strategy in Year 1, to help pupils learn basic maths skills, those with specific needs or difficulties in maths often stop using this technique to solve problems.

Topic

Algebra/Geometry

Learning Outcomes

Pupils use visual aids to support their reasoning, allowing them to solve more complex problems and equations.

How does it work?

Before they begin solving problems, pupils should be aware of what type of visual representation to create and use for a given mathematics problem. Pupils that are more proficient may already know how to do this, whereas this may need to be explained to pupils who struggle with maths,



due to a learning difficulty for example. Without a clear and methodical explanation on creating and using visual representations, these pupils may create visual representations that are not well presented, or contain incorrect or partial information.

The transition from physical objects or visual representations to using abstract equations can be difficult for some pupils. One strategy that teachers can use to help pupils transition from physical objects and visual representations to abstract equations is the Concrete-Representational-Abstract (CRA) framework.

This framework helps pupils to gain a conceptual understanding of a mathematical process, rather than just completing the algorithm (e.g., $2 + 4$, $2x + y = 27$); connecting concrete objects or visual representations to an abstract equation is a way to scaffold a pupil's understanding. The components of the framework are as follows:

Concrete: Pupils use three-dimensional objects to assist their learning, such as algebra tiles or manipulatives that represent variables and units,

Representational: Pupils use two-dimensional drawings to represent problems. These pictures may be shown to them by their teacher, included in learning material or drawn by the pupils themselves.

Abstract: Pupils solve problems that include numbers, symbols, and words without any concrete or representational assistance.



Why is it a good practice?

The above activities initiate the development of:

- Critical thinking skills
- Creativity
- Communication

Assessment

CRA is effective across all age levels and can assist pupils in learning concepts, procedures, and applications. Teachers should explain each component clearly and methodically, continually monitoring the pupils' work to assess their understanding and asking them questions about their thought processes and providing clarification as needed. The concrete and representational activities must reflect the actual process of solving the problem so that pupils are able to apply the process when solving an abstract equation. The below illustration highlights each of these components.

Inclusion

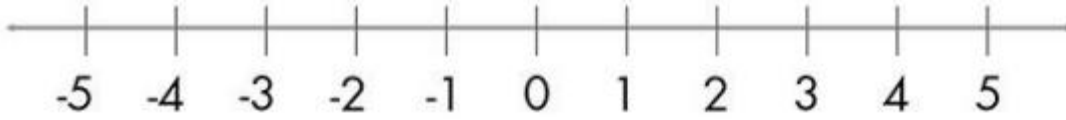
The activities allow pupils to mix and interact with each other.

Resources

<https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p05/#content>

Language

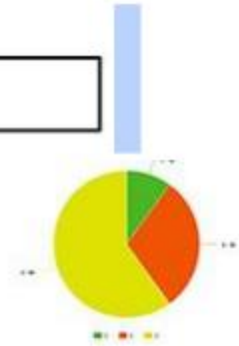
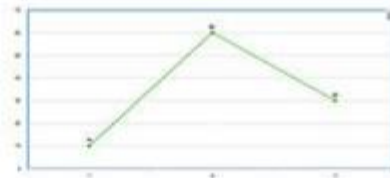
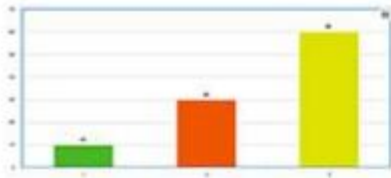
English



1/3

1/3

1/3





Learning Mathematics with Emerging Methodologies (Gamifying e.g., Kahoot)

The games are intended to support pupils' numeracy and digital literacy skills with new ways of engaging.

Topic

Algebra, Geometry, Quadratics (target age group 12-16 year-olds: United Kingdom Years 8 - 11 / Spain ESO 1 - 4)

Learning Outcomes

The games included in this section are designed to support and improve pupils' digital, numeracy and linguistic skills.

How does it work?

Pupils can play each game independently by pupils or with their teacher, which provides them with a flexible way to learn and teach. The games facilitate whole class teaching and/or opportunities for independent study within or outside of the classroom.

Why is it a good practice?

Kahoot! bills itself as a "game-based digital learning platform" that is fun for pupils. The games aim to provide pupils with new ways of engaging with technology and the learning material.



Assessment

“Because pupils and their instructor see immediate feedback between questions, this is a powerful tool for formative assessment; pupils hardly notice they are evaluating their own knowledge and being evaluated” (Lipp, G. 2015).

<https://learninginnovation.duke.edu/blog/2015/07/kahoot-as-formative-assessment/>

Inclusion

Resources

Maths | Resources | Junior Cycle for Teachers (JCT):

<https://www.jct.ie/maths/resources>

Curto Prieto, M. et al (2019, 12 marzo). Student Assessment of the Use of Kahoot in the Learning Process of Science and Mathematics.

MDPI Open Access Journals. Retrieved from:

<https://www.mdpi.com/2227-7102/9/1/55>

Language

English



“Realistic Maths Education (RME)” (e.g., Project IMaT – Inclusive Mathematics Teaching)

RME builds deep and long-term mathematical understanding by working with topics that are relevant to the pupils that in turn help them to make sense of intuitive strategies, such as matching different objects. With this, pupils gain ownership of their knowledge and a deeper understanding of the origins of maths.

Topic

Numbers/ Proportional Reasoning/Geometry/Algebra

Learning Outcomes

The major benefit of RME is that pupils can use flexible models to make sense of problems, rather than relying on (mis-)remembered rules.

Outcomes:

- The development of informal strategies to solve problems.
- Positive effects on pupils’ engagement and understanding.

How does it work?

Classes are taught with the help of an animated PowerPoints and printouts to guide lesson discussion.



These materials are designed so that pupils can generate their own strategies for dealing with mathematical problems. This means that a large part of the teacher's role is to support them in the process of mathematisation, encouraging pupils to engage as mathematicians by:

- Articulating their own reasoning clearly,
- Listening to others' strategies and explanations and looking carefully at mathematical representations.

Why is it a good practice?

RME classrooms promote a culture of listening, observing, and refining maths skills by incorporating diagrams and drawings to facilitate class discussions and aid understanding.

The materials encourage pupils to think first, solve later. That means that the materials are designed to support fundamental skills, such as problem solving and mathematical reasoning, while developing a deep understanding of key concepts and lasting proficiency.

Assessment

Within the modules and lessons there are questions to assess pupils' progress and share strategies. For example, in module Number 1 (N1) Fair Sharing, question B14 is especially useful to assess learning. Exams can also be based on this material as part of formal assessment.

Inclusion



Resources

“Realistic Maths Education” {(e.g. project IMaT – Inclusive Mathematics Teaching: Understanding and developing school and classroom strategies for raising attainment)}:

<https://uni.oslomet.no/imat/>

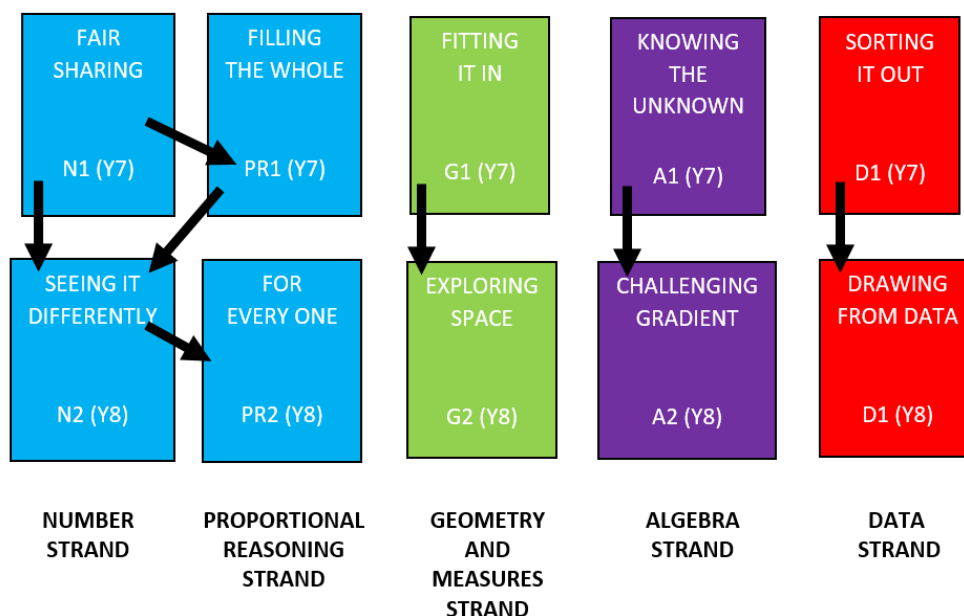
About: <https://uni.oslomet.no/imat/about-realistic-mathematics-education/>

Realistic Maths Education (Manchester Metropolitan University):

<https://rme.org.uk/our-materials/>

Language

English



The image above shows the 10 modules split into five topic strands and spread across two years.



Here is an example (materials and video): Proportional Reasoning 1 (PR1):
[Proportional Reasoning 1 \(PR1\) - Realistic Maths Education \(rme.org.uk\)](https://www.rme.org.uk/proportional-reasoning-1-pr1)



Quadrilaterals – Circumference and Area

This method involves constructing triangles and quadrilaterals using a computer or by hand, measuring required elements, using the appropriate units and calculating the circumference and area of geometric shapes composed of triangles and quadrilaterals.

Topic

Measuring (Shapes and Space)

Learning Outcomes

The lesson will give the opportunity to:

- Solve and apply linear equations.
- Construct triangles, analysing their rules and properties.
- Select and recalculate the appropriate units of measurement.
- Measure the angles and calculate the area of triangles and quadrilaterals.
- Construct quadrilaterals, analysing their rules and properties.



How does it work?

Pupils draw shapes composed of different geometric figures (mainly quadrilaterals, but they could use triangles or parts of circle) and then calculate their area and circumference.

There is also a project for pupils to complete at home. Discussions are held in class about calculating area and circumference, appropriate units of measurement, area of geometric shapes composed of parallelograms, rectangles, squares, rhombus, trapezoids, and triangles.

Why is it a good practice?

Pupils decide how and when to undertake their tasks. They can choose between constructing shapes by hand with geometric accessories and/or on a computer (a chance to improve their ICT skills).

Assessment

Evaluation grid (in Croatian): https://carnet-my.sharepoint.com/personal/marina_grubelic_skole_hr/_layouts/15/onedrive.aspx?id=%2Fpersonal%2Fmarina%5Fgrubelic%5Fskole%5Fhr%2FDocuments%2Fkriteriji%20%C4%8Detverokut%20projekt%2Epdf&parent=%2Fpersonal%2Fmarina%5Fgrubelic%5Fskole%5Fhr%2FDocuments&ga=1

Inclusion

Pupils draw/construct their own shapes, which can be simple or complex. Everyone works in his or her own time and at his or her own tempo.



Resources

<https://www.thinglink.com/scene/1323968786419679235>

<https://express.adobe.com/page/OHLSDvPVfkkdK/>

Language

Croatian



Fermi Problems

Fermi problems do not have an exact answer and are a good way to encourage pupils to think mathematically and make reasonable assumptions and estimates in order to arrive at an approximate solution. The more assumptions they make, the more accurate the estimate.

Topic

Numbers

Learning Outcomes

Pupils will learn to:

- Use a numerical expression in a set of natural numbers with zero and model a problem situation.

How does it work?

Activity 1

Divide pupils into teams and share out the tasks to be solved that are listed on the worksheet.

Tasks:

1. How many pupils would we need to equal the mass of an elephant?
2. How many bars of chocolate would we need to put together to make one that is 100m long?
3. Imagine you have a pile of £1 coins as high as Mt. Everest. How much would it be worth?



4. How long would it take to count to a million?

5. How many pizzas does our class eat in a year?

For each task, pupils write the solution and working out. After solving them as a team, they present their solutions to the rest of the class.

Activity 2

Pupils present their answers and their working out. Teams compare results and comment on the possible solutions.

Activity 3

As homework, pupils make up similar questions using them and their friends.

Activity 4

Pupils evaluate their performance. They explain whether they worked well as a team and if they would like to solve more problems using this method.

Why is it a good practice?

It encourages pupils to think mathematically and make accurate estimations. They solve problems by breaking them down into parts and determining the order in which they need to be solved, explaining their working out at the end.

Assessment



Inclusion

Tips for teachers to make these activities more inclusive:

- Explain each step to the pupils in detail and ensure that they are clear on what each task involves.
- Keep pupils' attention by using different teaching methods that encourage active participation.
- Minimise distractions by choosing a quiet, tidy working environment.
- Avoid noise and disruption from other pupils.
- Praise pupils for completing the tasks and keep a record of those who have applied themselves well.
- Ignore spelling mistakes or misspellings.
- Read the instructions for the written exam, check that the pupil has fully understood the instructions/tasks and take account of subsequent corrections in the exams.
- Use cream-coloured paper if possible and avoid red and green print.
- In order to be dyslexia-friendly, text should be written in Sans Serif font with at least 14 pt. Use bold or highlighted letters and avoid underlining titles or sequences of words, as this could cause words to merge visually. Increase the spacing between letters and use double spacing to separate lines. Break the text into smaller units and divide it into individual lines, rather than as continuous prose.



Resources

<https://innovativeteachingideas.com/blog/an-excellent-collection-of-fermi-problems-for-your-class>

https://navajomath.math.ksu.edu/wp-content/uploads/2015/03/fermi_questions_handouts_and_lesson_plan.pdf

Language

English



Probability

Activities to help pupils understand the probability of events with equal or different probability, the probability of opposite events, estimating probability and comparing experimental and theoretical probability.

Topic

Working with data, statistics, probability

Learning Outcomes

The pupils will learn to:

- Find and categorise all possible outcomes (which are mutually exclusive) for single and two successive events.
- Compare experimental and theoretical probabilities.
- Repeat experiments, which can lead to different results each time.

Pupils will usually arrive at a result that corresponds to the theoretical probability.



How does it work?

Activity 1: Turn the Disc (10 minutes)

On the *Flippin 'Discs* website (maths.org), the teacher spins the green and red discs and asks pupils to predict the probability. The teacher shows the scoreboard after 10 throws. Pupils win when both discs shown are the same colour; the number of wins divided by the number of throws gives the relative frequency.

Activity 2: Who gets there first? (20 minutes)

For this activity, you will need a worksheet, twelve counters and two regular dice.

Each counter represents a total shown by the dice (1-12). Participants take turns throwing the dice and moving the counter that corresponds to the number shown a step forward.

Encourage a class discussion by asking pupils questions such as: which figure do you think will reach the goal first and why? Can number 1 win?

At the end of the activity and after a class discussion, the table with all possible outcomes when rolling the dice is available online at *Two Dice Possibility Space* (transum.org). If you do not have access to the internet, you can create a similar table on paper.



Activity 3: "5 Tasks in 5 Minutes"(5 minutes)

Pupils are given a worksheet with five tasks to solve.

Why is it a good practice?

This activity allows pupils to develop:

- Critical thinking skills
- Communication skills
- Teamwork
- Creativity

Assessment

The activity contain questions that allow pupils to draw their own conclusions and explain why and how something is happening.

The assessment is based on the worksheet: "Five Tasks in 5 Minutes"

Inclusion

Inclusive Activity: "Choosing Sweets"

Pupils receive an opaque bag or box of 12 red, 6 blue and 3 green Sweets and a worksheet on which they should write down which colour sweet they remove from the box. Discuss the questions on the worksheet based on the results that the pupils have written down.

Resources

<https://www.transum.org/software/SW/SnailRace/PossibilitySpace.asp>

<https://nrich.maths.org/6123>

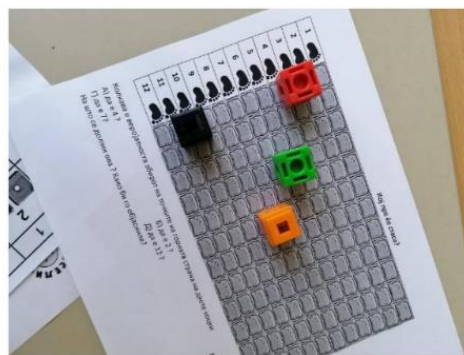


Language

English

Two Dice Possibility Space

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						





Project Task in School Project “Sustainable Tourism “

The pupils went to the tourist office and collected data on the number of tourists in Vodice, Croatia since data was first recorded. They then made line and bar charts, calculated percentages and analysed the data in groups.

Topic

Working with data, statistics, probability

Learning Outcomes

The pupil will learn to:

- Calculate percentages and apply a percentage account.
- Organise and analyse data displayed in a bar graph showing relative frequency.
- Display data in tables and line and bar graphs.

How does it work?

1. Pupils go to the tourist office to collect data.
2. They research tourism in their town.
3. They then analyse the data obtained and make graphs, that show for instance: temperatures in tourist season, the number of foreign and local visitors, the years with most/least guests and any reasons for this or a forecast of next year’s tourist season.



Why is it a good practice?

The activities in this section will help pupils to develop:

- Sustainable thinking skills.
- The ability to apply maths to their daily lives.
- The ability to work well as a team.

Assessment

Assessment grid:

<https://drive.google.com/file/d/1vaf7SvInGkEfdiUECnLgrzhg3DIymB8B/view>

Inclusion

All pupils can be involved in collecting data as a group. Handling real data encourages pupils to work together as a team.

Resources

<https://carnet->

my.sharepoint.com/:w:/g/personal/marina_grubelic_skole_hr/EfwNGRyNif5Em9OVCmC8bhYBGYgYM2AO2Xm8vA2lxLMpBA?e=zCxr8F

Language

Croatian



References

- Butterworth, B. & Kovas, Y. (2013). Understanding Neurocognitive Developmental Disorders Can Improve Education for All. *Science*, 340(6130), 300-305. Retrieved from: <https://doi.org/10.1126/science.1231022>
- Schleicher, A. (2019). PISA 2018: Insights and Interpretations. OECD. Retrieved from: [PISA 2018 Insights and Interpretations FINAL PDF.pdf \(oecd.org\)](#)



Co-funded by
the European Union



INSTITUT
JULIO ANTONIO 