



E = MD²

**E=MD2: Izvršnost u matematičkom
obrazovanju u inkluzivnom razredu kroz
e-debatu i raznolikost**

Ebook





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Uvod

Znanstvenici već dugo koriste matematiku za opisivanje fizičkih svojstava svemira. Neki su čak tvrdili da je svemir sam po sebi matematika, uključujući i ljudska bića. Kada je riječ o običnim ljudima, svjesni **smo sveprisutnosti matematike** u svakodnevnom životu, ali je **vrlo često ne koristimo na pravi način**. To se svaki dan događa i u učionici./

Najvažnije obrazovno razdoblje u našem životu je između 4. i 15. godine. U tom razdoblju učenici stvaraju mehanizme učenja, izgrađuju znanja i razvijaju osnovne vještine. Međutim, rezultati međunarodnih testova znanja i vještina ne pokazuju nužno da škole pomažu učenicima u stjecanju i razvoju ovih osnovnih vještina. Prema rezultatima PISA testiranja **matematičkih vještina 2018.**, učenici iz **Španjolske, Cipra, Rumunjske i Hrvatske postigli su minimalnu razinu (2. razina)**. **Učenici iz Sjeverne Makedonije nisu dosegli ni ovu osnovnu razinu**. Štoviše, rezultati nacionalnih testova diljem Europe počinju pokazivati razorni učinak nedavne pandemije COVID-19 na uspjeh školske djece, osobito one najugroženije.

Najveći **razlog za zabrinutost su oni neangažirani ili slabiji učenici** koji će, prema Schleicheru (2019), nestati na marginama društva bez pravog obrazovanja. Mnogi akademski radovi i globalna izvješća kao što je izvješće mreže Eurydice "Matematičko obrazovanje u Europi: zajednički izazovi i nacionalne politike" ukazuju na važnost motivacije i angažmana učenika. U



svjetlu ovih okolnosti osmišljen je projekt **“E=MD2: Izvrsnost u matematičkom obrazovanju u inkluzivnom razredu kroz e-debatu i raznolikost”** s ciljem podizanja razine matematičkih vještina europskih učenika, posebno onih s matematičkim poteškoćama (diskalkulija, disgrafija, matematička anksioznost...), povećavajući motivaciju za učenje i čineći njihovo iskustvo učenja manje pasivnim. /

S tim u vezi, radni tim projekta E=MD2 smatra da učenike treba pitati gdje su problemi u pristupu ili razumijevanju matematičkih sadržaja. Popularna metoda uključivanja ljudi u razgovor je matematička debata. **Ovom metodom ljudi sudjeluju u razmjeni ideja i stajališta o učenju matematike.** Rezultat je bolje razumijevanje međusobnih iskustava. U tom kontekstu, radni tim E=MD2 pitao je učenike, učitelje i roditelje za njihova mišljenja o poučavanju i učenju matematike./

Između rujna i prosinca 2022., tim E=MD2 proveo je niz anketa i online debata u različitim europskim zemljama. S jedne strane, **177 učitelja, 274 učenika i oko 100 roditelja iz Španjolske, Cipra, Rumunjske, Hrvatske i Sjeverne Makedonije** sudjelovalo je u internetskoj anketi u kojoj su opisali i realnost i izazove učenja i poučavanja matematike. S druge strane, **roditelji (20) i učitelji (20) iz dviju škola u Rumunjskoj i Sjevernoj Makedoniji sudjelovali su u internetskoj debati kako bi razmijenili iskustva i razvili bolje razumijevanje izazova s kojima se učenici susreću u školi i kod kuće kada se bave matematičkim aktivnostima.** To je učinjeno u nadi da će se pružiti bolja podrška i motivacija učenika da poboljšaju svoj uspjeh. Ovaj je dokument stoga sažetak nalaza anketa i rasprava./

Konačno, kao dio misije traženja izvrsnosti u podučavanju matematike u inkluzivnoj učionici, temeljni dio ovog procesa je podizanje profila



nastavnika matematike. U tu svrhu, E=MD2 tim sastavio je **zbirku najboljih praksi u nastavi matematike tako da su uključeni u novu E=MD2 nastavnu metodu** i interaktivnu [e-MATH DEBATE platformu](#) kreiranu i za učitelje i za učenike (i roditelje)./



Zaključci online ispitivanja

Učitelji

Kako je cilj projekta E=MD² otkriti gdje leže problemi u vezi s poučavanjem i razumijevanjem matematičkih sadržaja, prva metoda dobivanja podataka bila je online anketa u kojoj je sudjelovalo više od 500 ljudi, među kojima su bili i učitelji. Za ovu skupinu pitanja su osmišljena kako bi se saznalo o **njihovoj stvarnosti** u učionici i **njihovim percepcijama** kurikulumu, **njihovim stavovima** prema učenicima i stavovima učenika prema matematici./

Ovo su najznačajniji nalazi:

Većina učitelja teme iz kurikulumu matematike koje predaje smatra prikladnima (64,73%). Međutim, velika većina njih tvrdi da **kurikulum sadrži previše sadržaja (76,27%), zbog čega ne mogu odvojiti vrijeme za potrebe učenika s teškoćama u učenju (68,93%) i/ili posvetiti vrijeme darovitim učenicima (69,49%).** Što je još važnije, većina učitelja u uzorku (79,66%) navela je da smatra da je potrebno imati **više sati matematike tjedno**. To jasno daje do znanja da rade pod velikim vremenskim pritiskom i vrlo sažetim kurikulumom, što im otežava izvođenje lekcija na takav način da svi učenici razumiju i mogu primijeniti znanje.

Što se tiče njihove percepcije i percepcije njihovih učenika, gotovo svi anketirani učitelji slažu se da je matematika težak predmet (76,84%) i da **učenici ne vide smisao matematike (71,19%)**. Iznenađujuće otkriće također je da iako sudionici imaju **15 ili više godina iskustva u nastavi, samo mali dio njih je izjavio da se osjeća kompetentno podučavati matematiku učenicima s poteškoćama u učenju** (12,43% se osjeća kompetentno, a 40,68% se djelomično slaže s tvrdnjom da se osjeća kompetentno).

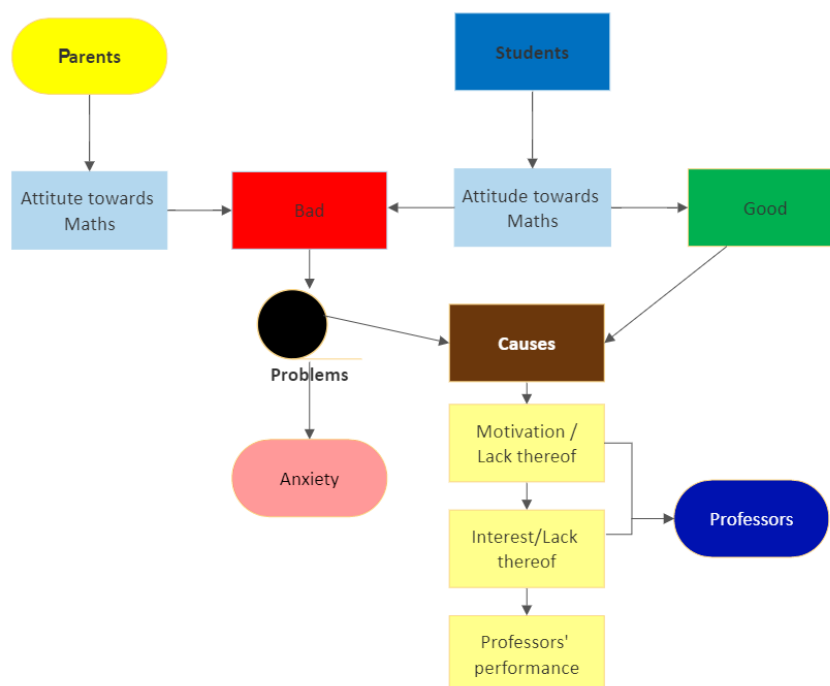


Unatoč svemu, jasno je da učitelji rade ono što mogu s onim što imaju. **Gotovo svi (98,03%) smatraju da je vršnjačko suradničko učenje korisna strategija** i priznaju da je podučavanje izvrstan način učenja (96,61%), praveći na taj način priliku za obje strane, nadarenim učenicima i učenicima s poteškoćama u učenju – pomoći i dobiti pomoć.

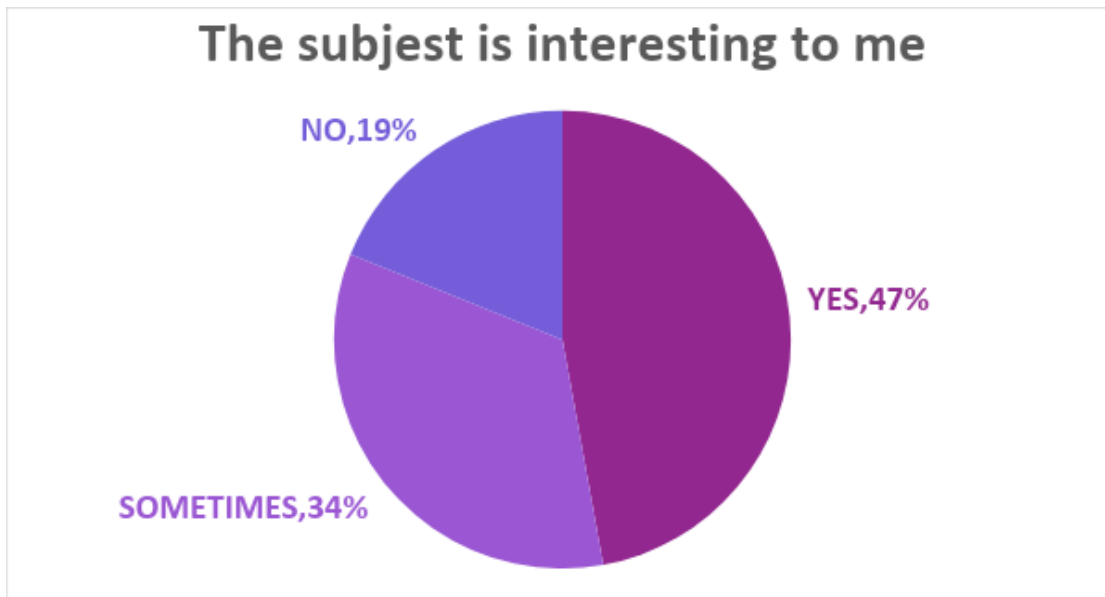
Iz ove ankete vidimo da su učitelji svjesni problema u podučavanju matematike, ali **nemaju vremena pomoći svojim učenicima i nemaju priliku unaprijediti svoje vještine**, što samo omogućuje da se ti problemi nastave.

Učenici

Radni tim projekta E=MD² je prije istraživanja imao sljedeću hipotezu. Ideja je bila da je na negativan stav učenika prema matematici snažno utjecao, s jedne strane loša učiteljska izvedba, a s druge strane roditelji. Kada su i stavovi i utjecaji bili negativni, to je dovelo do "matematičke anksioznosti". Na iznenađenje radnog tima, rezultati ankete imaju mnogo više nijansi nego što se prvotno pretpostavljalo.



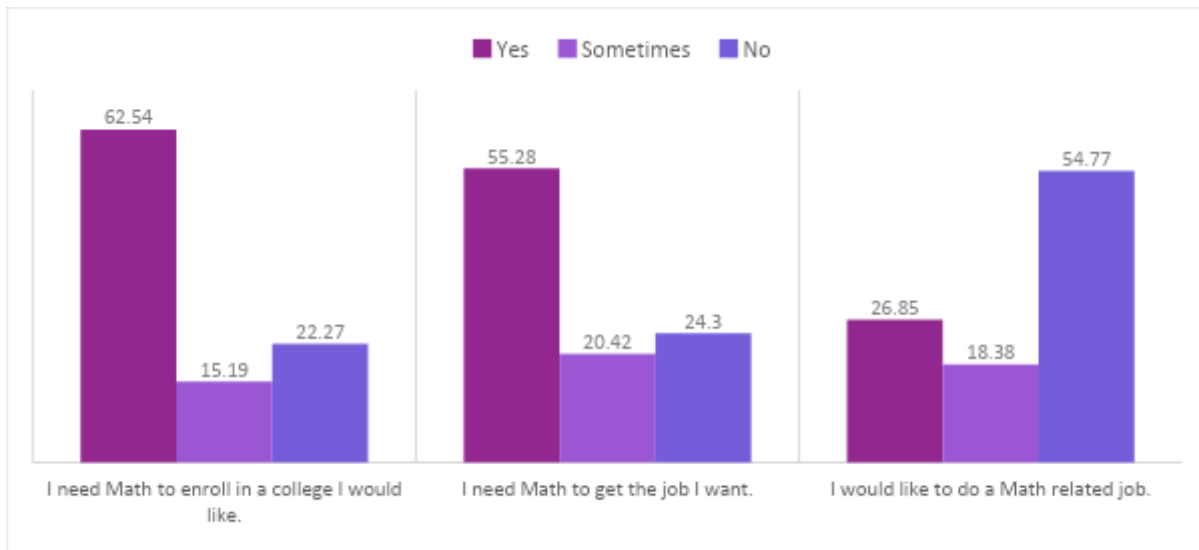
Neočekivano, od 274 ispitanika većina učenika se izjasnilo da **voli ili im se matematika sviđa (23,72% voli, a 48,9% sviđa im se)**, a samo 10,58% učenika izjavilo je da matematiku uopće ne voli. Ova prilično pozitivna tendencija pokazuje se i kada su učenici upitani bi li bilo "lijepo da nema nastave matematike" (58,42% se ne slaže). **Međutim, istraživanje također pokazuje veliki nedostatak interesa za matematiku među učenicima.** Na primjer, učenicima **niti su zanimljivi (50%) niti uopće korisni (27,01%) sadržaji** koje matematika poučava. Očito je da postoji podjela **50:50** između dvije strane.



Slika 5. Općenito mišljenje učenika o matematici. (Predmet me zanima - NE 19%, PONEKAD 34 %, DA 47%)

Što se tiče matematičke anksioznosti, **21,79%** učenika **složilo se** s tvrdnjom **"Matematika me čini nervoznim i zbunjenim"**, a ostalih 36,43% odgovorilo je "ponekad". Ovaj negativan osjećaj ogleda se i u činjenici da je **77% ispitanih učenika** odgovorilo s "da" ili "ponekad" na sljedeću tvrdnju: **"Nakon pismenog ispita osjećam da sam puno griješio čak i u zadacima koje sam znao"**.

Ne smijemo zanemariti ni **ulogu učitelja u matematičkom obrazovanju** jer oni imaju veliki utjecaj na radnu okolinu. Na primjer, **između 40 % i 50 % svih ispitanika smatra da ima opuštenu radnu atmosferu**, a da je disciplina glavni razlog tome. Samo 15% učenika ne osjeća se opušteno zbog radne atmosfere i discipline u nastavi. Očito je da prisutnost discipline u razredu jasno korelira sa stažom nastavnika.



Slika 9. Općenito mišljenje učenika o potrebi matematike u kasnijem životu.

Sada, iako učitelji matematiku doživljavaju kao težak predmet, oni ulažu vrijeme u pronalaženje zanimljivih nastavnih metoda kako bi učenike upoznali s gradivom. Jedna od tih metoda je vršnjačko učenje. Do sada se **37,46%** ispitanih učenika složilo s tvrdnjom „**Volim kada mi vršnjaci objašnjavaju zadatke, tako bolje učim**”, a **45%** učenika reklo je da **uživa objašnjavati matematičke zadatke svojim kolegama i smatra da tako bolje uče**. Očekivano, učenici bolje uče u parovima te kroz rasprave i sudjelovanje u nastavnom procesu.

Kako pokazuje anketa, učitelji matematiku doživljavaju kao težak predmet pa **traže pomoć onih učenika koji imaju interesa za matematiku** i koji nemaju poteškoća s razumijevanjem. To su oni koji **djeluju kao učitelji i pomažu drugim učenicima** razumjeti gradivo. Međutim, **još uvijek postoji mnogo zbunjenosti i tjeskobe u vezi s matematikom** kada učenici **moraju sami rješavati matematičke probleme**, na primjer na ispitu. To rezultira nedostatkom motivacije, što se prevodi u opću nezainteresiranost za matematiku kod gotovo polovice učenika diljem Europe. **Nadalje, ovi rezultati ne uzimaju u obzir činjenicu da, prema istraživanju, 2 ili 3 učenika**



u svakom razredu imaju poteškoće u učenju (Butterworth, B. & Kovas, Y., 2013.), učenici koji ne dobivaju toliko potrebnu osobnu pozornost od strane nastavnika zbog svog već pretrpanog nastavnog plana i programa i ograničenja vremena nastave.

Roditelji

Što se tiče roditelja, opće je mišljenje da je matematika teška i da učenici nisu dovoljno motivirani. Naime, od 169 ispitanika njih **61,57% priznalo je da njihovo dijete ima problema s matematikom**. Kao razloge roditelji navode motivaciju i nerazumijevanje materije. Naime, **40% učenika nije motivirano, a gotovo 31% ne razumije predmet** prema mišljenju roditelja./

Na pitanje može li **njihov stav prema matematici utjecati na stav djeteta, mišljenja su podijeljena gotovo 50:50** između onih koji u to vjeruju i onih koji ne vjeruju. Međutim, percepcija je kod hrvatskih roditelja koji tvrde da njihov stav nema utjecaja na stav njihova djeteta, dok su rumunjski, makedonski i španjolski roditelji suprotnog mišljenja./

Na pitanje što bi moglo pomoći njihovom djetetu da bude motiviranije za učenje matematike, odgovor je u osnovi bio da bi **matematika trebala biti zanimljivija, da bi trebalo biti više "gamifikacije" u nastavi matematike**. Štoviše, bilo je važno da učenici vide povezanost sa svakodnevnim životom i potrebom za matematikom. Ovo mišljenje je izraženo i u online debatama s učiteljima. Zaključci izvučeni iz ove rasprave razmatraju se u sljedećem odjeljku ovog dokumenta.

Online debate za razmjenu iskustava

Kao što je ranije spomenuto, radni tim E = MD² krenuo je u stjecanje boljeg razumijevanja izazova s kojima se učenici susreću u školi i kod kuće kada se bave matematičkim aktivnostima. U tu svrhu organizirane su **2 online debate za razmjenu iskustava**.

3. studenog 2022. grupa **učitelja (10) i roditelja (10)** iz **Sjeverne Makedonije** sastala se na internetskoj debati. Debatu su moderirali djelatnici škole **OOU „ILINDEN“** Kriva Palanka. Dana 12. studenog 2022. održana je ista debata, ovaj put u srednjoj školi **"Ion Agârbiceanu"** u **Alba Iuliji u Rumunjskoj**. I ovdje je u debati sudjelovalo **10 učitelja matematike i 10 roditelja**. U obje debate roditelji i djelatnici škole raspravljali su o mogućim načinima na koje bi mogli podržati i motivirati učenike da poprave svoj uspjeh.

Zaključci izvučeni u Sjevernoj Makedoniji i Rumunjskoj su sljedeći:

Iako matematika prvenstveno uči učenike kako razmišljati, daje im samopouzdanje i otvara mnoge mogućnosti za budućnost, **izuzetno zgusnut kurikulum** (u Rumunjskoj vrlo teoretski) ne dopušta učiteljima da **sadržaji poučavanja budu praktični**. Nadalje, broj sati matematike tjedno **ne dopušta** učiteljima rad s učenicima koji imaju **veće sposobnosti** ili **posebne obrazovne potrebe**.

Prema riječima roditelja i učitelja, **nezainteresiranost** za matematiku **dolazi do izražaja kako učenici odrastaju**, jer u početku vole matematiku. Posljedično, **kako matematika postaje sve složenija, manje su neovisni i manje sigurni u svoje sposobnosti**. Ovaj problem postaje vidljiv kada rade



domaću zadaću i trebaju odgovore na svoja pitanja. Često dolazi do frustracije kada im roditelji pokušavaju pomoći i pogrešno ih usmjeravaju.

Opći zahtjev roditelja prema učiteljima je da se **matematika učenicima učini opipljivijom**, tj. učenici ne razumiju korisnost matematike jer je toliko apstraktna. Stoga žele **više praktičnih vježbi koje se mogu prevesti i biti korisne u svakodnevnom životu.**

Annex 1 – Primjeri dobre prakse [en]

Linear Programming for Gifted Students ages 12-14

The purpose of this learning plan is to help pupils understand some of the differences between linear algebra and linear programming, using graphs of first-degree functions and solving inequalities. Then they practice by considering exercises, including ones with real life applications.

Topic

Algebra

Learning Outcomes

This Learning plan provides activities that lead/ enable/ support a pupil (mainly a gifted one)

- To identify the best connections between the first-degree function and the graphical representation of the function

- To identify the real solution to a minimum or maximum problem
- To identify the graphical solution of a minimum or maximum problem.
- To calculate the intersections of the graph with the coordinate axes.



How does it work?

The learning plans provides ideas for activities that suggest to the pupils:

- To explore the lines of connection between the field of definition of the function and the graphic representation.
- To look for the shortest connections.
- To specify what conditions are expected.
- To clarify whether the graph provides solutions for

determining the required minimum or maximum (optimized) quantities.

- To graphically represent the conditions in the hypothesis of the problem.
- To use their newly acquired knowledge to determine the maximum or minimum value of the function.
- To explore how to determine the requirements of the problem with the help of a graph

Why is it a good practice?

It encourages critical thinking, creativity and communication, and the content is connected to the real world.



Assessment

The learning plan suggests that pupils use the Internet to find similar or more advanced real-world problems and attempt to solve them using linear programming.

This emphasis on real world problems adds value to the organisation of the INNOMATH project for the Mathematics Meets Industry day, which reflects the expected impact of this activity in particular, as well as for the other activities proposed in the project.

Inclusion

The approach is addressed to the needs of gifted pupils.

Resources

7th grade maths textbook, problem collections:

<https://www.khanacademy.org/math>

Language

English



Services Assessment in an Organization

The Learning Plan provides for activities for pupils that are involved in a real research process with application in services evaluation. Basic issues and stages of the research process are taught, from the formulation of the problem and the goal to the final presentation of the results and conclusions.

Topic

Statistics (Research Methodology)

Learning Outcomes

This Good Practice provides a Learning plan with activities that lead/ enable/ support a pupil to acquire skills for

- Data Collection Methods and Sampling Methods and Techniques
- Construction and use of appropriate questionnaires (printed or electronic)
- Considering Questionnaire validity and reliability control methods (use of appropriate software)
- Using Methods of statistical analysis and presentation of

results (use of appropriate software)

- Presentation of results - Writing a detailed research report

How does it work?

The learning process is based on involving pupils and their teachers in a real-life process of evaluating the services of a community service provider that would be of interest to the pupils. The outcome will be a review or improvement of those services, for the benefit of the service provider (s.p.) and for the benefit of the pupils or citizens who use those services. The importance of the outcomes is in itself a great motivator. On the other hand, the skills acquired through research are also very important for the 21st century citizen. These skills are acquired through contacts - consultations with the representatives of the institution providing the services, but also through teamwork in preparing questionnaires (printed and electronic), data collection and input, data analysis, presentation of results and drawing conclusions. Throughout the process there is continuous discrete teacher support from teachers and evaluation, feedback for the results at each stage.

Why is it a good practice?

It encourages critical thinking, creativity and communication, and the content is connected to the real world. Moreover, it provides opportunities for debate and discussion and the use of digital means.

Assessment

Feedback and evaluation are continuous, from the preparation of the relevant questionnaires to the conduct of the research, analysis and presentation of the results.

Inclusion

The whole approach provides opportunities that are appropriate for pupils of any ability, thus it can be used in an inclusive class.

Resources

The theoretical framework will be taught in the classroom. However, the pupils will process the questionnaires (printed or electronic), the data entry, the statistical analysis and the preparation of the presentation of the results in the computer laboratory (with the support of the teachers).

Resources, Tools, Material, Attachments, Equipment

Important factors for evaluating Services

<https://www.fotoinc.com/news-updates/3-ways-to-evaluate-your-services>

<https://www.qualtrics.com/blog/how-to-measure-service-quality/>

Data Collection Methods and Sampling Methods and Techniques

<https://www.slideshare.net/swatiluthra5/sampling-ppt>

<https://www.slideshare.net/7mukut/sampling-techniques-49115431>

<https://www.slideshare.net/Indraneeltu/icfai-ib>

Construction and use of appropriate questionnaires (printed or electronic)

<https://www.slideshare.net/deepthisreenivas1/questionnaire-design-in-research>

<https://mopinion.com/top-21-best-online-survey-software-and-questionnaire-tools-an-overview/>

Questionnaire validity and reliability control methods (use of appropriate software)

<https://www.youtube.com/watch?v=Odlhhtg-3LE>

<https://www.youtube.com/watch?v=pVfByfoQ1IU>

Methods of statistical analysis and presentation of results (use of appropriate software)

Presentation of results – Writing a detailed research report



Co-funded by
the European Union



Language

English



Deriving formulas and determining the area of 2D shapes drawn on a grid of dots

The Learning Plan provides for activities for pupils that are involved in a real research process with application in services evaluation. Basic issues and stages of the research process are taught, from the formulation of the problem and the goal to the final presentation of the results and conclusions.

Topic

Algebra

Learning Outcomes

At the end of the lesson, the learner will be able:

- To express a variable through other variables in a given equation
- To performs simple formulas
- To determines the area of 2D shapes drawn on a grid of dots
- To use formulas from maths and other subjects
- To present concise, substantiated arguments to explain solutions or generalizations using: symbols, diagrams, or graphs
- To develops a sense of cooperation and empathy with classmates.

How does it work?

The lesson lasts 40 minutes.

Activity 1: There is a discussion about the preconceptions about the area of regular shapes and how to find the formula for the area of this shape formed by joining a rectangle and two semicircles and shows the model of the figure that has been prepared in advance. The pupils explain their answers that the shape is formed by merging a rectangle and two semicircles, which can be seen by cutting the parts and merging the semicircles and getting a circle. The area is the sum of the areas of the rectangle and the circle and then they write the formula for area of the form

$$P = w^2 \cdot \pi / 4 + x \cdot w$$

Activity 2: The teacher gives an activity in which the pupils in pairs look for a connection between the area of shapes drawn on a grid with dots, the points in the form and the points on the perimeter) i.e., examine the formula for area A, on the shapes drawn on a grid with dots (with points in the form and p points on the perimeter) (Peak theorem).

The teacher asks questions:

How do you write down your findings?

Do you notice any patterns?

Can you find a general rule?



It is concluded that the area of the form is one less than the sum of the points in the form and half of the points on the perimeter are and the formula is derived

$A = I + p / 2 - 1$ known as the Pick's theorem.

Pupils in pairs draw shapes on dots and then count the dots in the form and the dots on the perimeter of the form and calculate their area using the Pick formula in the same activity using a geometric board.

The obtained solutions are discussed. Couples self-assess.

Pupils in pairs use the GeoGebra app to draw shapes and then count the points in the form and the points on the perimeter of the form and calculate the area with the Peak formula and then check the answer in the area calculation menu.

Activity 3: In the school yard, a shape is made with the help of a rope and boats and then the points in the shape and the points on the perimeter of the shape are counted and its area is calculated using the Peak theorem.

Why is it a good practice?

The above activities initiate the development of:

- critical thinking
- cooperation

- creativity
- communication

Assessment

By developing these skills, pupils are trained to learn on their own. The first activity allows to diagnose the pupils' knowledge of the topic and according to the obtained results to plan the future activities. The activity with Peak theorem enables connection with real situations, develops critical thinking, increases communication skills, creativity, and cooperation between pupils. The use of digital tools to validate acquired knowledge provides rapid feedback for both pupils and the teacher.

Inclusion

The teacher can include elements that the pupil can manipulate such as: elastic shapes on a geometric board. (A 2d shape model made of coloured paper).

Resources

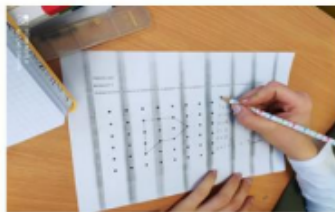
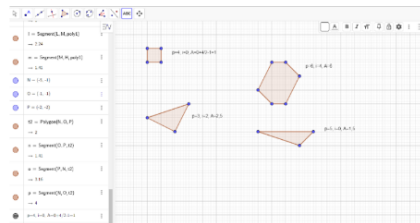
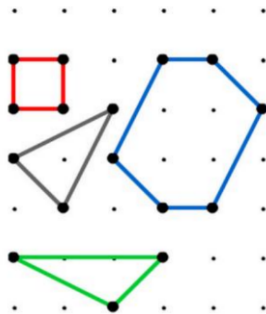
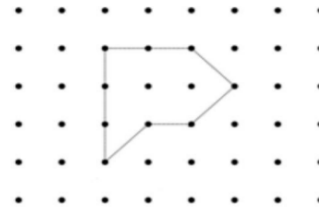
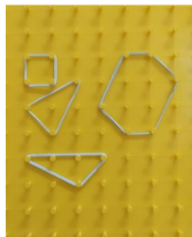
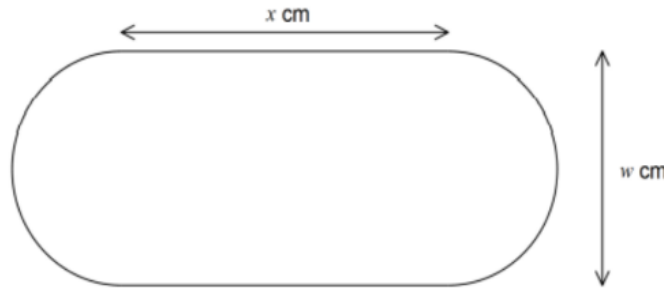
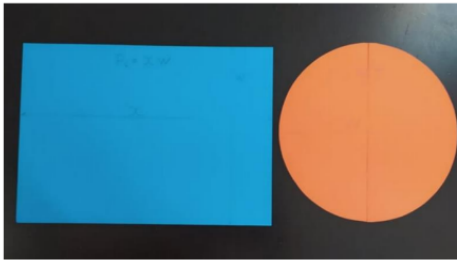
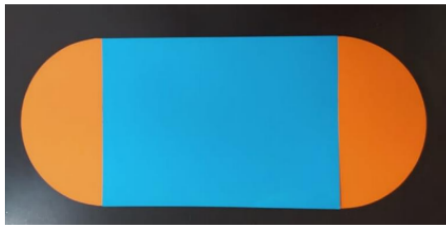
<https://nrich.maths.org/content/id/8506/1>

<https://nrich.maths.org/1867>

<https://www.geogebra.org>

Language

English





Percentages

Pupils should learn to calculate a percentage of a given whole and solve problems in the context of percentage increase and decrease. The lesson is to expand the "Percentage" section that is taught in the first semester. It is realized in 3 school hours in a duration of 120 min.

- School class 1 (40 min): Diagnosis of previous knowledge
- School class 2 (40 min): Expanding the term percentage with percentage
- increases and decreases
- School class 3 (40 min): Reflection

Topic

Percentages

Learning Outcomes

At the end of the lesson, the learner will be able:

- To use simple fractions and percentages to compare quantities (visualization forms)
- To create and solve problems with percentage increase and decrease
- To research and use various strategies in solving text problems
- To develop a sense of cooperation and empathy with classmates

How does it work?

School Class 1: Diagnostics for calculating a percentage of a number

Fold the shape by x percent (5 min)

In this short activity the pupils are divided into 4 groups and each group receives a different worksheet. The worksheets show the different shapes that pupils should divide according to the instructions. The goal is to visually connect a part of a whole as a fraction and as a percentage and comparison.

Percentage cards (35 min)

With this activity the pupils practice calculating a percentage of a given whole (number). Each pupil receives a card on which one task is written and one solution to a task from another card. The game is started by a pupil who reads the task from his card. The others calculate and the pupil who has the solution to that task shows the card and then reads his task. The game continues in the same way until all the tasks from the cards are read.

School class 2: Expanding the term percentage with percentage increases and decreases

Bookshop–Presentations (40 min)

Pupils are divided into 4 groups and two bookshops are set up. Two of them will be buyers, and two groups will be sellers.

Benches are set up as stands for sale and several textbooks, notebooks, pencils, rulers for sale are arranged on them. Groups of sellers determine

the price of those products and whether their price will increase by a certain percentage or decrease. The price and the percentage increase and decrease are recorded and placed next to each product. Purchasing pupils choose one product at a time and need to calculate how much it will cost after the increase / decrease. Then the groups change their roles.

Why is it a good practice?

The above activities initiate the development of critical thinking, communication, collaboration, and creativity. By developing these skills, pupils are trained to learn independently.

Assessment

The first activity (School class 1) allows to diagnose pupils' knowledge of the topic and according to the results obtained planning future activities. The activity with bookshops (School class 2) allows connection to real situations, develops critical thinking, increases communication skills, creativity and collaboration among pupils.

The use of digital tools to verify adopted knowledge provides quick feedback for both pupils and teacher.

School class 3: evaluation

Game Genially Self-evaluation (15 min)

Description: Pupils on their tablets or smartphones in pairs solve a quiz in Genially for percentage increase and decrease.

The quiz is designed so that pupils can immediately see where they went wrong and have a short discussion about the mistakes made.

Quiz in H5P-Quiz (20 min)

Pupils solve a **quiz on H5P** that is set up on learning platform. From there, information about the points scored in the quiz is automatically obtained. If it is not possible on the platform the quiz can be placed on a website only



in that case the pupils need to send their answers to the teacher.
<https://ucimatematika.weebly.com/good-practice-percent.html>

Impressions (5 min)

A short discussion about the realization and the achieved success of the **quiz on H5P**.

- **Formative evaluation** by monitoring pupil activity throughout all three hours and the self-evaluation with a **quiz on Genially**.
- **Summative evaluation** is done through the points won on **the H5P quiz**.



Inclusion

Activity1: Figures drawn with overlap by 50%, 25%, 10%,20% and similar

assessment made by the teacher.

Activity 2: Lego cubes that are given a value of 5% and 10% for assembling 3D forms

Resources

Shapes for percentage of numbers & Percentage cards:

<https://ucimatematika.weebly.com/active-learning.html>

Percentage of numbers-Quiz in Genially:

<https://view.genial.ly/620226b64cdd0a00183fbe5d/game-percentage-of-number>

Language

English and Macedonian

50% of 16	35	20% of 85	8
25% of 160	17	1% of 100	40
1% of 200	1	25% of 184	2
5% of 140	46	5% of 100	7
20% of 55	5	20% of 115	11
10% of 120	23	20% of 0	12
50% of 96	0	25% of 148	48
5% of 200	37	10% of 160	10
25% of 60	16	15% of 20	15
5% of 180	3	25% of 140	9



Sale Discount 5% *попуст*

Sale discount 20% *попуст*

Sale discount 15% *попуст*

Sale discount 25% *попуст*

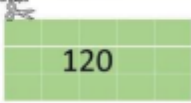
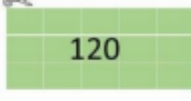
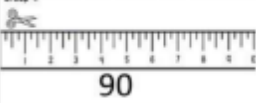
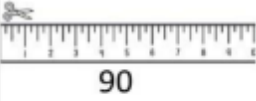

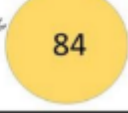
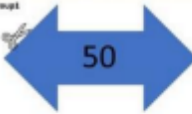

Sale discount 75% *попуст*

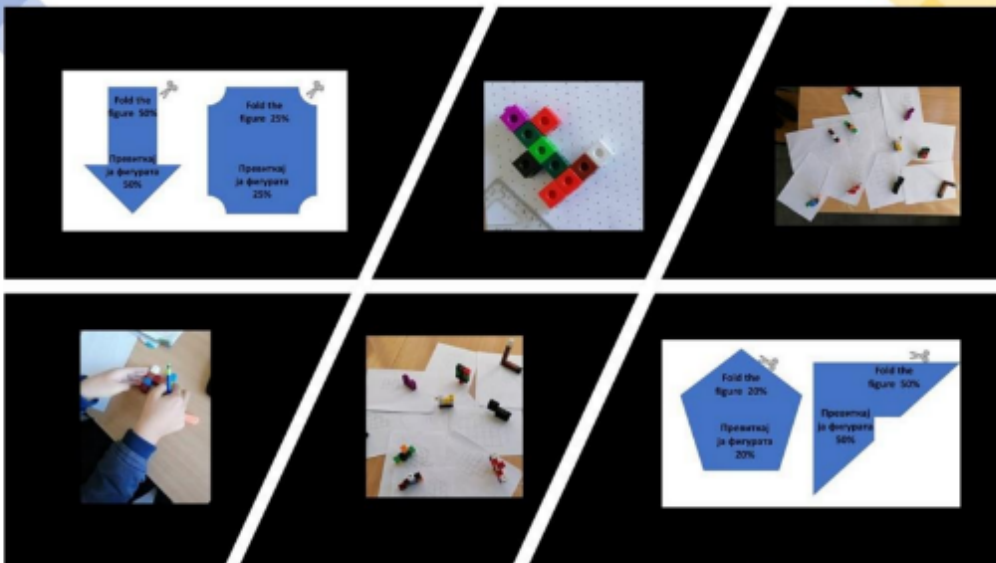
higher prices 22% *повисоки цени*

higher prices 44% *повисоки цени*

higher prices 25% *повисоки цени*



<p>Group 1</p>  <p>By folding show $\frac{1}{5}$</p> <p>If the figure is number 120, what is its value after folding?</p> <hr/>  <p>By folding show 20%</p> <p>If the figure is number 120, what is its value after folding?</p>	<p>Group 4</p>  <p>By folding show $\frac{1}{10}$</p> <p>If the figure is number 90, what is its value after folding?</p> <hr/>  <p>By folding show 10%</p> <p>If the figure is number 90, what is its value after folding?</p>
<p>Group 2</p>  <p>By folding show $\frac{1}{4}$</p> <p>If the figure is number 84, what is its value after folding?</p> <hr/>  <p>By folding show 25%</p> <p>If the figure is number 84, what is its value after folding?</p>	<p>Group 3</p>  <p>By folding show $\frac{1}{2}$</p> <p>If the figure is number 50, what is its value after folding?</p> <hr/>  <p>By folding show 50%</p> <p>If the figure is number 50, what is its value after folding?</p>



Learn functions and discuss how equations and inequations can be solved using GeoGebra representation

Solving inequations, especially non-linear ones, can result in a long and difficult process. Using GeoGebra pupils can represent non-linear functions and they can easily interpret the regions where the functions are positive or negative and consequently, they can solve the inequation.

Topic

Algebra and functions

Learning Outcomes

The games developed within this project are designed to both support and improve the digital, numeracy and linguistic skills

How does it work?

Pupils start drawing non-linear functions in GeoGebra and they can discuss and find out how they change. Then the teacher gives an activity in which the pupils have to represent paraboles and they have to find a connection between the form and:

- The term of highest degree is positive or negative.
- Changing the independent term.

Teacher can help pupils asking:

- Do the branches of the parable go up or down? Is there any relation to the changes in the expression of the function?

- Do you notice any patterns?
- Can you find a general rule?

Next step is solving inequations using the representation of the parable and analysing where it is positive or negative.

Finally, they discuss the solutions obtained within the group and with other groups.

Why is it a good practice?

The above activities initiate the development of:

- creativity
- communication
- critical thinking
- cooperation

Assessment

The activities develop critical thinking, increases communication skills, creativity, and cooperation between pupils. The use of digital tools to validate acquired knowledge provides rapid feedback for both pupils and the teacher.

Inclusion

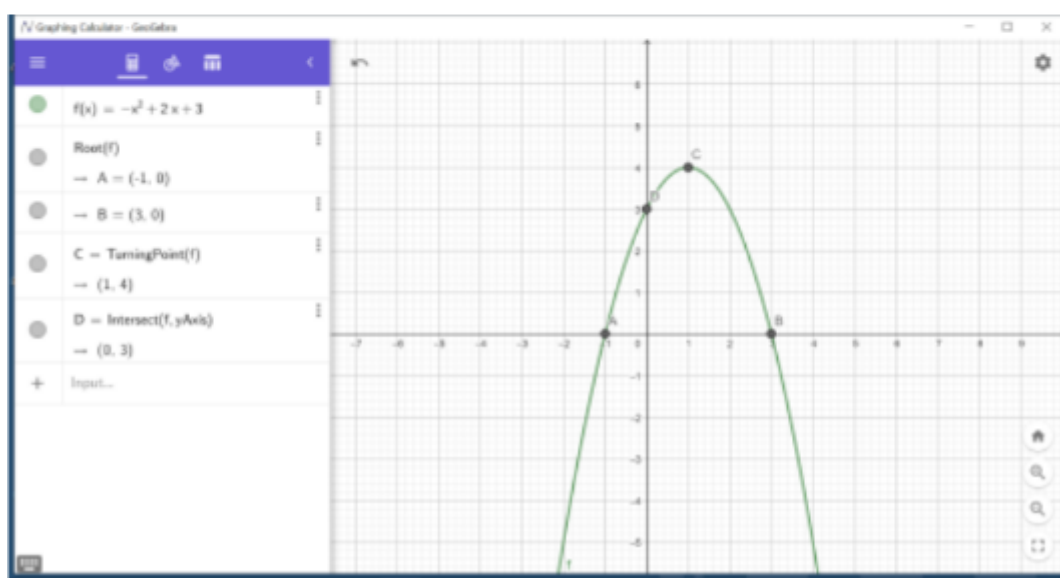
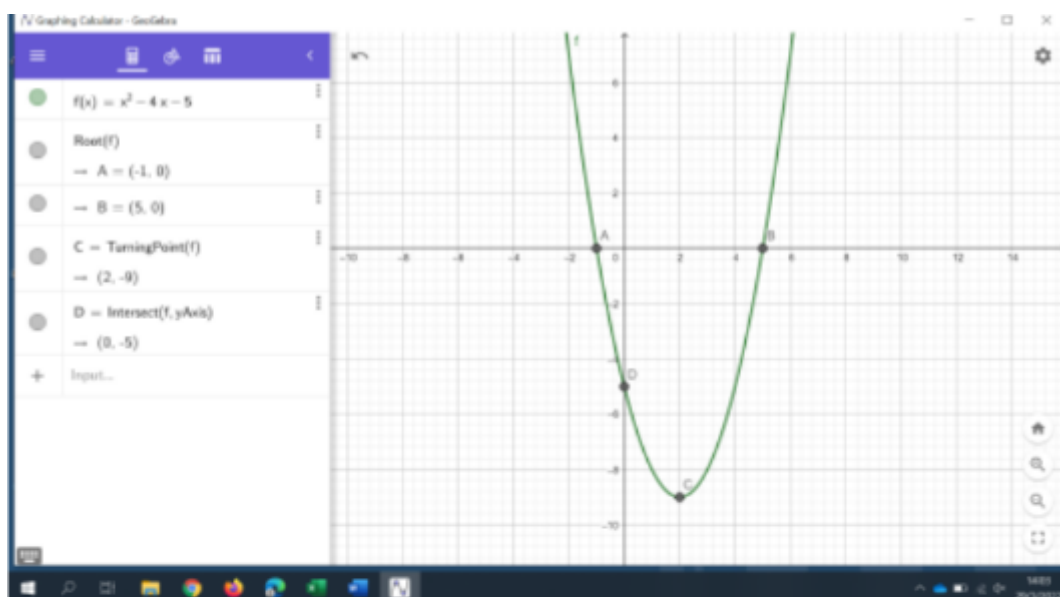
The activities allow for the formation of heterogeneous groups.

Resources

<https://www.geogebra.org/>

Language

English



Learn how to operate integer numbers (addition, subtraction, multiplication, and division) with different variations seeing the different properties.

Sometimes pupils have problems with negative numbers or with the different properties of the operations with integer numbers. Using that quiz

pupils can practice at different levels of difficulty, and they can self-correct and see where they have more problems.

Topic

Numbers

Learning Outcomes

The games developed within this project are designed to both support and improve the digital, numeracy and linguistic skills.

How does it work?

Teacher post in google classroom a link with different exercises and a deadline of one week. The exercises can be personalized and at different levels, so all the exercises can fill the differences between children.

Pupils can work in their houses with the exercises, and they can do it as many times as they want. The goal is to see the mistakes and try to not repeat and improve every time they do.

Before the deadline, pupils must make a screen capture and post in google classroom for evaluation. Teacher will see in google classroom how many pupils did the job, their results, and their mistakes, and decide to put the focus on the most common mistakes and prepare another quiz.

Pupils have one week to complete every quiz. They must upload a screen shot of their results in the educational platform. The evaluation is not quantitative (from 0 to 10). The pupils think that it is a more visual and understandable way of working.

Why is it a good practice?



The activities initiate the development of:

- critical thinking
- creativity
- communication

Assessment

The use of digital tools to validate acquired knowledge provides rapid feedback for both pupils and the teacher.

Inclusion

The exercises can be personalized and at different levels, so all the exercises can fill the differences between children.

Resources

<https://www.thatquiz.org/>



Language

English

t h a t q u i z Teachers: login or [create an account](#) | or [\[search\]](#) or [\[learn more\]](#)

Login/Email Password

integers	fractions	concepts	geometry
Arithmetic	Identify	Time	Triangles
Inequality	Arithmetic	Money	Shapes
Averages	Inequality	Measure	Geometry
Exponents	Averages	Place value	Points
Factors	Simplify	Graphs	Angles
Algebra	Probability	Sets	Number line
Calculus			Trigonometry

vocabulary		geography	
English	Spanish	Americas	Europe
French	German	Africa	Asia

science			
Cells	Anatomy	Elements	Conversion

English Español Deutsch Français Italiano Português Català
 Euskara Magyar Polski Slovenščina Türkçe Ελληνικά Українська
 中文(简体) עברית

Have a test code?

Visual Representations

An evidence-based strategy to help pupils learn abstract maths concepts and solve problems is to use visual representations. More than just a picture or a detailed illustration, a visual representation – often called a schematic representation or schematic diagram – is an accurate description of the mathematical quantities and relationships of a given problem. The purpose of this visual is to reflect the pupil's understanding of the problem and to help him or her solve it correctly. For example, in the photo on the right, a pupil uses a visual representation – here, a pie chart – to learn about equivalent fractions. Even though teachers implement this strategy in the first grade to help pupils learn basic maths facts, pupils with difficulties and difficulties in learning mathematics often do not continue to use it alone to solve problems.

Topic

Algebra / Geometry

Learning Outcomes

Students learn to reason symbolically, and consequently the complexity and type of equation and problem they can solve increases.

How does it work

Before they can solve problems, however, pupils must first know what type of visual representation to create and use for a given mathematics problem. Some pupils—specifically, high-achieving pupils, gifted pupils—do this automatically, whereas others need to be explicitly taught how. This is especially the case for pupils who struggle with mathematics and those with mathematics learning disabilities. Without explicit,

systematic instruction on how to create and use visual representations, these pupils often create visual representations that are disorganized or contain incorrect or partial information.

A move from concrete objects or visual representations to using abstract equations can be difficult for some pupils. One strategy teacher can use to help pupils systematically transition among concrete objects, visual representations, and abstract equations is the Concrete-Representational-Abstract (CRA) framework.

The Concrete-Representational-Abstract (CRA) framework helps pupils gain a conceptual understanding of a mathematical process, rather than just completing the algorithm (e.g., $2 + 4$, $2x + y = 27$). Systematically connecting concrete objects or visual representations to the abstract equation is a way to scaffold a pupil's understanding. The components of the framework are:

Concrete – Pupils interact and manipulate three-dimensional objects, for example algebra tiles or other algebra manipulatives with representations of variables and units.

Representational – Pupils use two-dimensional drawings to represent problems. These pictures may be presented to them by the teacher, or through the curriculum used in the class, or pupils may draw their own representation of the problem.

Abstract – Pupils solve problems with numbers, symbols, and words without any concrete or representational assistance

Why is it a good practice?

The above activities initiate the development of:

- critical thinking
- creativity
- communication

Assessment

CRA is effective across all age levels and can assist pupils in learning concepts, procedures, and applications. When implementing each component, teachers should use explicit, systematic instruction and continually monitor pupil work to assess their understanding, asking them questions about their thinking and providing clarification as needed. Concrete and representational activities must reflect the actual process of solving the problem so that pupils are able to generalize the process to solve an abstract equation. The illustration below highlights each of these components.

Inclusion

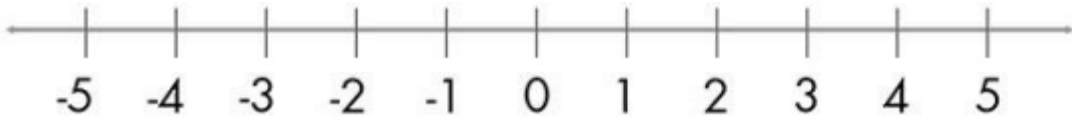
The exercises allow the formation of heterogeneous groups.

Resources

<https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p05/#content>

Language

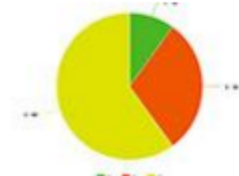
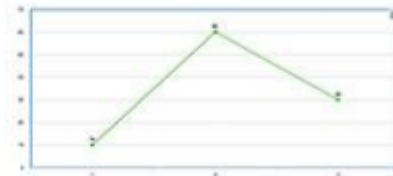
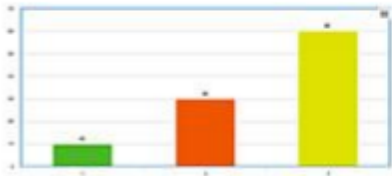
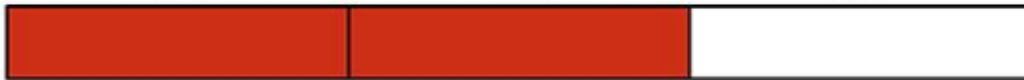
English



1/3

1/3

1/3



Learning Mathematics with Emerging Methodologies (Gamifying e.g., Kahoot)

The games are intended to support pupils' numeracy digital literacy skills with new ways of engaging.

Topic

Algebra, Geometry, Quadratics (Target population, 12-16-year-olds: United Kingdom Grades 8 - 11 / 1 - 4 ESO Spain)

Learning Outcomes

The games developed within this project are designed to both support and improve the digital, numeracy and linguistic skills.

How does it work?

Each game can either be played independently by pupils or with a teacher in a classroom setting. This is intended to provide greater flexibility of use, both for teachers and their pupils. The games facilitate whole class teaching and/or opportunities for independent study within or outside the classroom.

Why is it a good practice?

Kahoot! bills itself as a “game-based digital learning platform,” but all pupils see is game. The games are intended to provide pupils with new ways of engaging with the material and with technology.

Assessment

“Because pupils and their instructor see immediate feedback between questions, this is a powerful tool for formative assessment; pupils hardly notice they are evaluating their own knowledge and being evaluated” (Lipp, G. 2015).

<https://learninginnovation.duke.edu/blog/2015/07/kahoot-as-formative-assessment/>

Inclusion

Resources

Maths | Resources | Junior Cycle for Teachers (JCT) -
<https://www.jct.ie/maths/resources>

Curto Prieto, M. et al (2019, 12 marzo). Student Assessment of the Use of Kahoot in the Learning Process of Science and Mathematics. MDPI Open Access Journals. Retrieved from:
<https://www.mdpi.com/2227-7102/9/1/55>

Language

English

“Realistic Maths Education (RME)” {e.g., project IMaT – Inclusive Mathematics Teaching}

RME builds deep and long-term mathematical understanding by working from contexts that make sense to pupils. By helping them make sense of their intuitive strategies –like visually aligning caps and umbrellas–. Through this process, pupils gain ownership of their mathematical knowledge and a deep understanding of where mathematics comes from.

Topic

Numbers / Proportional Reasoning / Geometry / Algebra

Learning Outcomes

The major benefit of RME is that pupils can use flexible models to make sense of problems, rather than relying on (mis-)remembered rules.

Outcomes:

- Develop informal strategies to solve problems.
- Positive effects on pupils’ engagement and understanding.

How does it work?

The classes are carried out with the help of an animated PowerPoints to guide lesson discussion, as well as a printable Activity Sheets.

These materials are designed so that pupils can generate their own strategies for dealing with mathematical problems. This means that a large part of the teacher’s role is to support them in the process of mathematization, encouraging pupils to engage as mathematicians by: articulating their own reasoning clearly,

listening to others' strategies and explanations and looking carefully at mathematical representations.

Why is it a good practice?

RME classrooms promote a culture of listening, observing, and refining mathematical techniques.

RME classrooms emphasize diagrams and drawings so that discussions arise, and understanding is attained.

The materials have pupils think first, solve later. That means that the materials are designed to support fundamental skills like (problem solving and mathematical reasoning) while developing deep understanding of key concepts and lasting fluency.

Assessment

Within the modules and sessions there are questions to assess their progress and share strategies e.g. in module Number 1 (N1) Fair Sharing, question B14 is useful Assessment for Learning. Nonetheless, for a formal assessment it would be most suitable to draw up an exam from the material.

Inclusion

Resources

“Realistic Maths Education” {(e.g. project IMaT – Inclusive Mathematics Teaching: Understanding and developing school and classroom strategies for raising attainment)} – <https://uni.oslomet.no/imat/>

About:

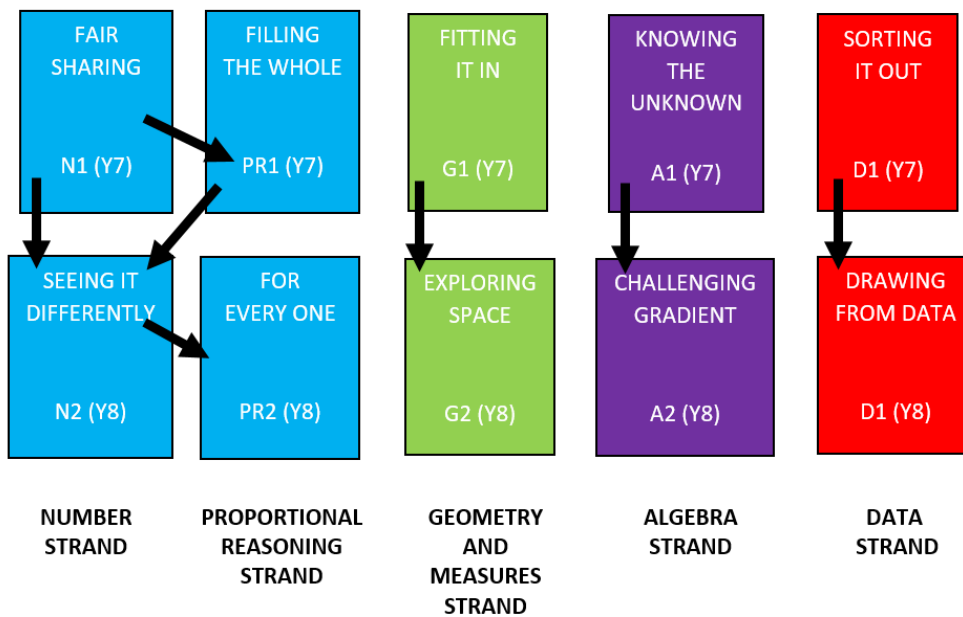
<https://uni.oslomet.no/imat/about-realistic-mathematics-education/>

Realistic Maths Education (Manchester Metropolitan University)-

<https://rme.org.uk/our-materials/>

Language

English



The image above shows the 10 modules split into 5 topic strands and across two 'years.

An example (materials and video): Proportional Reasoning 1 (PR1):
[Proportional Reasoning 1 \(PR1\) - Realistic Maths Education \(rme.org.uk\)](https://rme.org.uk)



Četverokuti – opseg i površina [Quadrilaterals – circumference and area]

The method includes construction of triangles and quadrilaterals using either geometric accessories or ICT, measuring required elements, using appropriate units and calculating circumference and area of geometric figures composed of triangles and quadrilaterals.

Topic

Measuring (Shapes and space)

Learning Outcomes

The lesson will give the learner the opportunity:

- To solve and apply the linear equation.
- To construct triangles, analyse their properties and relations.
- To select and recalculate the appropriate units of measurement.
- To calculate and apply the circumference and area of triangles and quadrilaterals and the measure of angles.
- To construct quadrilaterals, analyse their properties and relations.]

How does it work?

A task is given to pupils, they should draw some shapes and a figure composed of different geometric figures (mainly of quadrilaterals, but they could use triangles or parts of circle) and then calculate their circumference and area.

There is also a project assignment, and pupils do it at home. In class discussions are held about calculation of area and circumference, appropriate units, area of geometric figures composed of several basic ones).

Pupils are supposed to use parallelogram, rectangle, square, rhombus, trapezoid, and triangles.

Why is it a good practice?

Pupils decide how they will do their task. They choose time when to do it. They can choose between constructing with geometric accessories and/or ICT (so it is possible to improve their ICT skills).

Assessment

Evaluation grid (in Croatian):

https://carnet-my.sharepoint.com/personal/marina_grubelic_skole_hr/_layouts/15/onedrive.aspx?id=%2Fpersonal%2Fmarina%5Fgrubelic%5Fskole%5Fhr%2FDocuments%2Fkriteriji%20%C4%8Detverokut%20projekt%2Epdf&parent=%2Fpersonal%2Fmarina%5Fgrubelic%5Fskole%5Fhr%2FDocuments&ga=1

Inclusion

Pupils draw/construct their own ideas and figures, so they can be very simple or complex ones. Everyone works in their own time and tempo.

Resources

<https://www.thinglink.com/scene/1323968786419679235>



Co-funded by
the European Union



<https://express.adobe.com/page/OHLSDvPVfkkdK/>

Language

Croatian



Fermi Problems

Fermi problems are problematic mathematical problems that do not have an exact answer. They are executive for encouraging pupils to think mathematically and require them to make reasonable assumptions and estimates in order to arrive at an approximate solution. The more assumptions they make, the more accurate the estimate.

Topic

Numbers

Learning Outcomes

The learner will learn to:

- Use a numerical expression in a set of natural numbers with zero and model a problem situation.

How does it work?

Activity 1

Divide pupils into teams. Divide the tasks to be solved on the worksheet.

Tasks that pupils solve:

1. How many pupils together have a mass like an elephant?
2. How many packets of candy do we need to put together a 100m long candy bar?
3. Imagine you have a pile of 2 kuna coins like Mt Everest, how much would it be worth?
4. How long should it take to count to a million?
5. How many pizzas does our class eat in one year?

For each task, write the solution and the procedures for solving it, and the way you thought and solved the task. After solving within the team, you will present solutions to the whole class.

Activity 2

Pupils present their solutions and procedures, and present the assumptions they used in solving. Teams compare results and comment on the obtained solutions.

Activity 3

Ask pupils for homework to come up with a few questions for themselves to ask their friends as a new homework assignment.

Activity 4

Pupils complete a performance evaluation. They respond to how they have collaborated within it and whether they want us to work more often this way by solving tasks of this type.

Why is it a good practice?

Pupils need to think mathematically and make good assumptions. An important element of problem solving is the ability to break problems down into parts and determine the order in which they are solved. At the end they have to explain how they arrived at their solutions.

Assessment

Inclusion

Tips for teachers when implementing this good practice in an inclusive classroom:

Instruct the pupil in detail about each step and element of the mathematical activity and familiarise them with the structure of the tasks.

Explain the individual operations and the sequence.

Attract attention with different didactic tools and encourage active participation.

Minimise distractions (do not sit by the window or in the middle of restless pupils, but in a quiet place in the classroom, e.g. with quiet pupils or pupils with whom they work well, so that they are disturbed as little as possible by what is happening in class, the work area should only contain what is necessary for the work). The instructions given to him should be brief. Reward the perseverance of attention (the completion of a task is accompanied by praise, which is recorded in a notebook).

Pay no attention to spelling mistakes or misspellings. Read the instructions for the written exam, check that the pupil has understood the instructions/tasks well and take account of subsequent corrections in the exams. Use cream-coloured paper if possible and avoid red and green print. Increase the font size in comprehensible reading texts. Use sans serif letters for the text to be read by a pupil with dyslexia; the font size should be at least 14 pt. Use bold or highlighted letters. Avoid underlining titles or strings of words, which can cause words to blend visually. Increase the spacing between letters and lines and separate lines by double spacing. Break the text into smaller units and divide it into individual lines, not a continuous sequence.

Resources

<https://innovativeteachingideas.com/blog/an-excellent-collection-of-fermi-problems-for-your-class>



Co-funded by
the European Union



https://navajomath.math.ksu.edu/wp-content/uploads/2015/03/fermi_questions_handouts_and_lesson_plan.pdf

Language

English



Probability

Planned activities for understanding the term probability of events with equal and different probability, probability of opposite events, estimation of probability of. comparison of experimental with theoretical probability.

Topic

Working with data, statistics, probability

Learning Outcomes

The learner will learn to:

- Find and sort systematically all possible outcomes (which are mutually exclusive) for single events and two successive events
- Compare experimental with theoretical probabilities
- Recognise that the repetition of experiments can lead to different results
- Increase the number of times the experiment is repeated

Pupils will usually arrive at a result that corresponds to the theoretical probability.

How does it work?

(10 min) Activity 1 Turn the disc

On the Flippin 'Discs website (maths.org), the teacher spins the discs green and red and asks pupils to predict the probability. 10 throws are simulated and then the scoreboard is shown. A win is considered achieved when both discs are the same colour. The number of wins divided by the number of throws gives the relative frequency.

(20 min) Activity 2 Who gets there first?

A worksheet, a "Do not be bad man" figure and two playing dice with numbers from 1 to 6 are used.

Participants take turns turning the dice and moving the figure whose ordinal number corresponds to the sum of the dots on top of the dice.

Each figure moves 1 step. Appropriate questions are asked, such as: Which figure do you think will reach the goal first? And why? A discussion develops.

At the end of the activity and the discussion, the table with all possible outcomes when turning the two cubes is attached in the web version

Two Dice Possibility Space (transum.org). In the absence of suitable material (computer, internet, etc.), a table with the results can also be made on paper.

(5 min) "5 tasks in 5 minutes"

Pupils are given a worksheet with 5 tasks to solve.

Why is it a good practice?

Through the activities provided for class, pupils will develop their skills for:

- critical thinking
- communication
- collection
- creativity

Assessment

The activities contain questions that pupils have to answer independently and draw conclusions. Why? How? Why does what is happening happen?

They are expected to discuss and defend their views.

The assessment is based on the worksheet "Five tasks in 5 minutes"

Inclusion

Inclusive activity: "Pulling candy"

Pupils receive an opaque bag or box of 12 red, 6 blue and 3 green candies and a worksheet in which they should write down what kind of candy they pulled out of the box. According to the data from the table discussion of the questions from the worksheet.

Resources

<https://www.transum.org/software/SW/SnailRace/PossibilitySpace.asp>

<https://nrich.maths.org/6123>

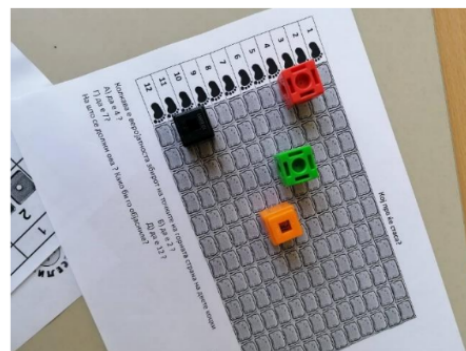
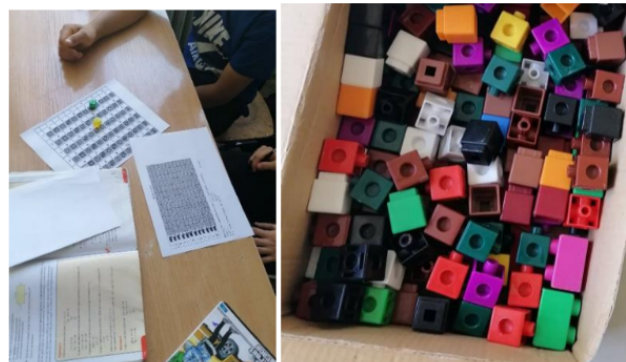
Language

English



Two Dice Possibility Space

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						





Project task in school project “Sustainable tourism “

The pupils went to the tourist office and collected data on the number of guests since the beginning of tourism in Vodice, Croatia (or at least data that was recorded). Then they made linear and bar charts, calculated percentages and analysed the data in heterogeneous groups.

Topic

Working with data, statistics, probability

Learning Outcomes

The learner will learn about/to:

- Calculate the percentage and apply a percentage account.
- Organize and analyse data displayed with diagram relative frequency bar graph.
- Display data in tabular form and linear and bar graph.

How does it work?

1. Go to the tourism office to collect data
2. research about tourism in our town
3. analyse the data
4. make graphs – temperature in tourist season, number of foreign and domestic guests, year with most/least guests and reasons for this, forecast for next tourist season

Why is it a good practice?

Through the activities provided for class, pupils will develop their skills for:

- sustainable thinking
- applying mathematics in daily life
- working in group (peer-to-peer work)

Assessment

Assessment grid:

<https://drive.google.com/file/d/1vaf7SvInGkEfdiUECnLgrzhg3DIymB8B/view>

Inclusion

Since group work and project tasks are involved, all pupils, including pupils with disabilities, are equally included in all activities. There is a lot of peer-to-peer cooperation and learning from experience with real data.

Resources

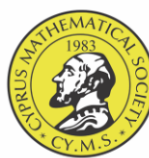
https://carnet-my.sharepoint.com/:w:/g/personal/marina_grubelic_skole_hr/EfwNGRyNif5Em9OVCmC8bhYBGYgYM2AO2Xm8vA2lxLMpBA?e=zCxr8F

Language

Croatian

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- Butterworth, B. & Kovas, Y. (2013). Understanding Neurocognitive Developmental Disorders Can Improve Education for All. *Science*, 340(6130), 300–305. Retrieved from: <https://doi.org/10.1126/science.1231022>
- Schleicher, A. (2019). PISA 2018: Insights and Interpretations. OECD. Retrieved from: [PISA 2018 Insights and Interpretations FINAL PDF.pdf \(oecd.org\)](#)



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